18. It frequently happens that a number of faces of a crystal intersect each other, or would intersect if produced till they met, in parallel lines. Such an assemblage of faces is called a 'zone.' The faces of a zone are all perpendicular to one plane, and their poles lie in a great circle, which will be called the 'zone-circle.' A line parallel to the intersections of the faces of a zone will be called the 'axis' of the zone.

19. Let *hkl*, *pqr* be the symbols of any two faces in a zone. Then, if

$$\mathrm{u} = kr - lq,\ \mathrm{v} = lp - hr,\ \mathrm{w} = hq - kp,$$

uvw will be the symbol of the zone containing the faces *hkl*, *pqr*, or of the zone-circle through the poles of *hkl*, *pqr*.

20. A face may be common to two zones, or its pole may be the intersection of two zone-circles.

Let hkl, pqr be the symbols of two zones. Then, if

$$u = \mathrm{kr} - \mathrm{lq},\ v = \mathrm{lp} - \mathrm{hr},\ w = \mathrm{hq} - \mathrm{kp},$$

uvw will be the symbol of the face common to the zones hkl, pqr.

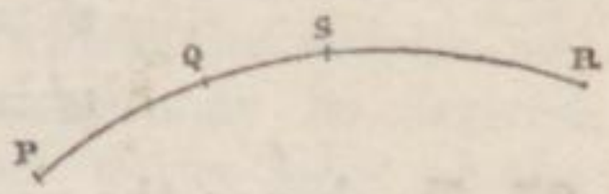
21. Let *uvw* be the symbol of a face in the zone uvw. Then,

$$u u + \mathrm{v} v + \mathrm{w} w = 0.$$

Any positive or negative whole numbers, including zero, which, when substituted for *u*, *v*, *w*, satisfy the above equation, are the indices of a face in the zone uvw; and any positive or negative whole numbers, including zero, which, when substituted for u, v, w, satisfy the same equation, are the indices of a zone containing the face *uvw*.

22. Let P, Q, R, S be four poles in one zone-circle, PR being larger than PQ, and PQ, PR, PS measured in the same direction from P. Let their symbols be,—

FIG. 7.



$$\mathrm{P}\ efg,\ \mathrm{Q}\ hkl,\ \mathrm{R}\ pqr,\ \mathrm{S}\ uvw.$$

$$\frac{[\mathrm{PS}]}{[\mathrm{SR}]} = \frac{fw - gv}{vr - wq} = \frac{gu - ew}{wp - ur} = \frac{ev - fu}{uq - vp},$$

$$\frac{[\mathrm{PQ}]}{[\mathrm{QR}]} = \frac{fl - gk}{kr - lq} = \frac{gh - el}{lp - hr} = \frac{ek - fh}{hq - kp}.$$

Then,

$$\frac{[\mathrm{PS}]}{[\mathrm{SR}]}(\cot \mathrm{PS} - \cot \mathrm{PR}) = \frac{[\mathrm{PQ}]}{[\mathrm{QR}]}(\cot \mathrm{PQ} - \cot \mathrm{PR}).$$