the upper or lower sign being taken according as T and D are measured from 111 in the same, or in different directions. Whence, m being known, h and k may be found.

128. In the form hkl, where $h + k + l = 0$, the distance between any two poles, not a multiple of 60°, being known, we can find the distance of one of them from the nearest pole of the form $2\bar{1}\bar{1}$. If this distance be θ, h, k, l may be found from the equations

$$\tan\theta = \sqrt{3}\frac{k-l}{2h-k-l}, \quad h+k+l=0.$$

129. If the distances between any pole of the form hkl and each of two other poles of the same form be given, the three poles not being in one zone-circle, the given distances, or their supplements, will be two of the arcs H, K, L, V.

$$\frac{\tan\theta}{\tan 60^\circ} = \frac{\tan\frac{1}{4}(\mathrm{K}-\mathrm{L})}{\tan\frac{1}{4}(\mathrm{K}+\mathrm{L})}, \quad \frac{\sin\theta}{\sin 60^\circ} = \frac{\sin\frac{1}{2}\mathrm{H}}{\sin\frac{1}{2}\mathrm{V}},$$

$$\frac{\tan\phi}{\tan 60^\circ} = \frac{\tan\frac{1}{4}(\mathrm{L}+\mathrm{H})}{\tan\frac{1}{4}(\mathrm{L}-\mathrm{H})}, \quad \frac{\sin\phi}{\sin 60^\circ} = \frac{\sin\frac{1}{2}\mathrm{K}}{\sin\frac{1}{2}\mathrm{V}},$$

$$\frac{\tan\psi}{\tan 60^\circ} = \frac{\tan\frac{1}{4}(\mathrm{K}-\mathrm{H})}{\tan\frac{1}{4}(\mathrm{K}+\mathrm{H})}, \quad \frac{\sin\psi}{\sin 60^\circ} = \frac{\sin\frac{1}{2}\mathrm{L}}{\sin\frac{1}{2}\mathrm{V}}.$$

Two of the four distances H, K, L, V being known, T and one of the angles θ, ϕ, ψ may be found; and then the indices may be found from the equations

$$\tan\theta = \sqrt{3}\frac{k-l}{2h-k-l}, \quad 2\tan\mathrm{T}\cos\theta = \frac{2h-k-l}{h+k+l}\tan\mathrm{D},$$

$$\tan\phi = \sqrt{3}\frac{l-h}{2k-l-h}, \quad 2\tan\mathrm{T}\cos\phi = \frac{2k-l-h}{h+k+l}\tan\mathrm{D},$$

$$\tan\psi = \sqrt{3}\frac{h-k}{2l-h-k}, \quad 2\tan\mathrm{T}\cos\psi = \frac{2l-h-k}{h+k+l}\tan\mathrm{D}.$$

130. To find the distance between any two poles.

Let P, Q be the poles of hkl, pqr; O, A the poles of 111, 100; OA = D. Let PQ meet the zone-circle through the poles of the form $10\bar{1}$ in M. Then, h, k, l, p, q, r being known, the symbol of M is known by (19), (20). tan MOA, tan POA, tan QOA are known by (117). Therefore POM, QOM are known. PO is given in terms of D by (117).