

## IV. Goniometrie und Trigonometrie

Winkel- funktionen	$\text{Sinus (sin)} = \frac{\text{Gegenkathete}}{\text{Hypotenuse}}$ $\text{Cosinus (cos)} = \frac{\text{Ankathete}}{\text{Hypotenuse}}$		$\text{Tangens (tg)} = \frac{\text{Gegenkathete}}{\text{Ankathete}}$ $\text{Cotangens (ctg)} = \frac{\text{Ankathete}}{\text{Gegenkathete}}$			
Beziehungen zwischen den Funktionen	$\sin^2 a + \cos^2 a = 1$ $\text{tg } a = \frac{\sin a}{\cos a}$ $\text{ctg } a = \frac{\cos a}{\sin a}$		$\sin a = \sqrt{1 - \cos^2 a}$ $\sin a = \frac{\text{tg } a}{\sqrt{1 + \text{tg}^2 a}}$ $\cos a = \frac{1}{\sqrt{1 + \text{tg}^2 a}}$		$\text{tg } a \cdot \text{ctg } a = 1$ $\text{tg } a = \frac{\sin a}{\sqrt{1 - \sin^2 a}}$ $\text{ctg } a = \frac{\cos a}{\sqrt{1 - \cos^2 a}}$	
Verlauf in den vier Quadranten (Vorzeichen)	Funktion	1. Quadrant	2. Quadrant	3. Quadrant	4. Quadrant	
	sin cos tg ctg	0 → +1 (+) +1 → 0 (+) 0 → +∞ (+) +∞ → 0 (+)	+1 → 0 (+) 0 → -1 (-) -∞ → 0 (-) 0 → -∞ (-)	0 → -1 (-) -1 → 0 (-) 0 → +∞ (+) +∞ → 0 (+)	-1 → 0 (-) 0 → +1 (+) -∞ → 0 (-) 0 → -∞ (-)	
Um- formungen	Funktion	-a	90° ± a	180° ± a	270° ± a	
	sin cos tg ctg	- sin a + cos a - tg a - ctg a	+ cos a ∓ sin a ∓ ctg a ∓ tg a	∓ sin a - cos a ± tg a ± ctg a	- cos a ± sin a ∓ ctg a ∓ tg a	
Komplement- winkel und Supplement- winkel	$\sin a = \cos (90^\circ - a)$ $\cos a = \sin (90^\circ - a)$ $\text{tg } a = \text{ctg } (90^\circ - a)$ $\text{ctg } a = \text{tg } (90^\circ - a)$		$\sin a = \sin (180^\circ - a)$ $\cos a = -\cos (180^\circ - a)$ $\text{tg } a = -\text{tg } (180^\circ - a)$ $\text{ctg } a = -\text{ctg } (180^\circ - a)$			
Additions- theoreme	$\sin (a \pm \beta) = \sin a \cos \beta \pm \cos a \sin \beta$ $\cos (a \pm \beta) = \cos a \cos \beta \mp \sin a \sin \beta$ $\text{tg } (a \pm \beta) = \frac{\text{tg } a \pm \text{tg } \beta}{1 \mp \text{tg } a \text{tg } \beta}$ $\text{ctg } (a \pm \beta) = \frac{\text{ctg } a \text{ctg } \beta \mp 1}{\text{ctg } \beta \pm \text{ctg } a}$		$\sin a + \sin \beta = 2 \sin \frac{a + \beta}{2} \cos \frac{a - \beta}{2}$ $\sin a - \sin \beta = 2 \cos \frac{a + \beta}{2} \sin \frac{a - \beta}{2}$ $\cos a + \cos \beta = 2 \cos \frac{a + \beta}{2} \cos \frac{a - \beta}{2}$ $\cos a - \cos \beta = -2 \sin \frac{a + \beta}{2} \sin \frac{a - \beta}{2}$			
Funktionen des doppelten und halben Winkels	$\sin 2a = 2 \sin a \cos a$ $\cos 2a = \cos^2 a - \sin^2 a$		$\text{tg } 2a = \frac{2 \text{tg } a}{1 - \text{tg}^2 a}$ $\text{ctg } 2a = \frac{\text{ctg}^2 a - 1}{2 \text{ctg } a}$			
	$\sin \frac{a}{2} = \sqrt{\frac{1 - \cos a}{2}}$		$\cos \frac{a}{2} = \sqrt{\frac{1 + \cos a}{2}}$			