omitted. This is the resistance given to the piston in its descent. The pistons of an engine for drawing water from deep mines must descend again by their own weight in order to repeat their stroke. This must require a preponderance on that end of the working-beam to which they are attached, and this must be overcome by the moving power during the effective stroke. It makes, therefore, part of the whole work to be done, and must be added to the weight of the column of water which must be raised.

This is very easily ascertained. Let the velocity of the piston in its descent be V, the area of the pump-barrel A, and the area of the piston-valve a. It is evident, that while the piston descends with the velocity V, the water which is displaced by the piston in a second is $(A—a)\,V$. This must pass through the hole of the piston, in order to occupy the space above, which is left by the piston. If there were no contraction, the water would go through with the velocity $\frac{A—a}{a}V$; but as there will always be some contraction, let the diminished area of the hole (to be discovered by experiment) be b; the velocity therefore will be $V\frac{A—a}{b}$. This requires for its production a head of water $\frac{V^2}{2g}\left(\frac{A—a}{b}\right)$. This is the height of a column of water whose base is not A but $A—a$. Calling the density of water d, we have for the weight of this column, and the force p is $d \times \overline{A—a} \times \left(\frac{A\ \ a}{b}\right)^2 \times \frac{V^2}{2g}, = \frac{d\,V^2\,(A—a)^3}{2\,g\,b^2}$. This we see again is proportional to the square of the velocity of the piston in its descent, and has no relation to the height to which the water is raised.

If the piston has a button-valve, its surface is at least equal to a; and therefore the pressure is exerted on the water by the whole surface of the piston. In this case we