

to $A A$. The Square of $6 \frac{1}{2}$ the Half of the Root is $42 \frac{1}{4}$; from which subtracting 32 , there remains $10 \frac{1}{4}$, the Root of which is $3 \frac{1}{4}$, a little less: Take that from $6 \frac{1}{2}$, and there remains 3 and $\frac{2}{4}$; add that to $6 \frac{1}{2}$, and it will make $9 \frac{1}{2}$; and these $3 \frac{1}{4}$ and $\frac{1}{2}$ will be the two Roots; which shews that the Water can never rise when the Barrel is empty, above 3 foot $\frac{2}{4}$ and a little more, tho' you play the Piston as long as you please; but if you had fill'd the Barrel 9 Foot $\frac{1}{2}$, you might make the Water rise 12 Foot compleat by several Strokes of the Piston.

LET us suppose now that the Barrel is 14 Foot up to the Piston, and that the Stroke of the Piston is 2 Foot; $32 - A$ will be to 32 , as $14 - A$ to $16 - A$. To find the Equation easily, you must multiply 32 by 2 , the Difference of 14 and 16 : The Product is 64 for the absolute Number, and that of $16 A$, will be the Number of the Roots, and $A A$ will be equal to $16 A - 64$; the Square of half the Root is 64 ; from whence subtracting 64 , there remains 0 , whose Root is 0 , which being taken from and added to 8 , still makes 8 ; which shews that there is but one Root, and that the Water can't rise above 8 Foot; but if you make the Piston play ever so little higher than 2 Foot, the Water will rise 14 Foot. The Analogy is easy; for the Piston being raised 2 Foot, the Barrel will be 16 Foot, and that Water being at 8 Foot, there will remain 6 Foot of Air; but 32 is to 24 the Complement of 8 Foot to 32 , as 8 Foot of rarified Air to 6 Foot of common Air; then the Water will raise no higher than 8 Foot, if the Piston plays but 2 Foot.

THENCE you see, that to draw up Water to a considerable Height, as 20 Foot, the Breadth of the Pump-Barrel must be diminish'd, and a sufficient Space must be allow'd for the Stroke of the Piston; for, supposing that the Surface of the Piston be 4 times broader than the Base of the Barrel, the rising of the Piston 1 Foot, will have the same Effect as if it rose 4 , if the Diameter of the Piston were only equal to that of the Barrel; if then the Stroke be a Foot and a half, it will be the same as if it rose 6 Foot, and were of the same Breadth: Now the 4 Terms of Equation

being $32 - A$; 32 , $20 - A$, $26 - A$, there will be 6 times 32 , viz. 192 for one Term of the Equation, and $26 A$ for the other, according to what has been said; there will be then $A A$ equal to $26 A - 192$; the Square of half the Roots is 169 less than 192 ; and consequently if you pump a long time, you may raise the Water 20 Foot.

IF in the Example above-mention'd, you take 8 Foot for the highest Term of the Water, when the Barrel is 14 Foot, and the Stroke of the Piston 2 Foot, 'tis easy to prove, that if you suppose 9 Foot of Water upon the Clack, it will continue to rise by the playing of the Piston 2 Foot; for there will remain 5 Foot of Air. Now there is a less Proportion betwixt 5 and 7 , than there is betwixt 27 , the Complement of 5 to 32 , and 32 , and consequently the Water will rise higher than 9 Foot. The Proportion will still be more unequal, if you take 10 or 11 Foot; and if you take 7 instead of 8 Foot, the Water will still rise, for there will remain 7 Foot of Air; now 25 , the Complement of 7 to 32 , is to 32 as 7 to $8 \frac{2}{3}$; then if the Piston goes 2 Foot, it will raise the Water higher than 7 Foot; it will rise still more easily, if you pour in only 6 Foot of Water; for there will be 8 Foot of Air. Now the Complement 26 is to 32 as 8 to $9 \frac{2}{3}$; then if instead of $9 \frac{2}{3}$, which makes the *Equilibrium*, the Piston goes 10 Foot, it will make the Water rise still better than when it was at 7 Foot; and better still than when it is at 5 Foot, &c. If you would know what Play the Piston must have to raise the Water 30 Foot, you must take a Number a little greater than the half of 30 , as 16 , at which Point pretty near the Water, will rise with the greatest Difficulty; the Complement is 16 , the Remainder of Air is 14 ; as 16 is to 32 , so is 14 to 28 . The Piston then must rise 14 Foot; or if the Barrel be 2 Inches Diameter, the Piston must be 7 Inches $\frac{1}{2}$; for the Square of $7 \frac{1}{2}$ is $56 \frac{1}{4}$, which is a little more than 14 times 4 the Square of 2 Inches; and then it will be sufficient that the Stroke of the Piston be one Foot; but as it is still more difficult at an Elevation of 18 Foot, the Piston must be 8 Inches Diameter, to raise the Water above 18 Foot, when its Stroke is but one Foot.

BOOK IV.