tried upon the subject in the Paris barracks near the Hotel de Ville (Caserne Napoléon); but owing to some defect in the arrangements the direction of the current was found sometimes to reverse itself, bringing back the vitiated air into the building instead of carrying it off; and the men had suffered from ophthalmia in consequence. He had not yet had any opportunity of applying to barracks the system of ventilation described in the paper. The importance of taking off the vitiated air at the bottom of the room, whereby alone it could be removed at once without polluting the atmosphere within the building, was sufficiently demonstrated by a visit to the basement of the Lecture Theatre in which they were now assembled; and in simply passing along quickly under the seats of the theatre when occupied by a number of persons, he had felt completely stifled by the poisonous atmosphere drawn off from the room. It was not surprising therefore that in large assemblies persons were frequently made ill by mere defect of ventilation; and in the case of hospitals it appeared not unreasonable to suppose that the very walls of the building must become impregnated with infection. These difficulties he considered could only be effectually obviated by the system described in the paper, ensuring the immediate removal of all the vitiated air by taking it off at the bottom of the room.—"*Proceedings Institution of Mechanical Engineers.*"

THE STABILITY OF DOMES.

By E. Wyndham Tarn, M.A., Architect.

(*With an Engraving.*)

The few writers who have attempted to treat the subject of the Equilibrium of Domes mathematically, have entirely failed to obtain results that are of any practical use to the architect. Their failure has arisen from taking a too theoretical view of the subject, and endeavouring by mathematical reasoning to find the *form* which a dome ought to have in order that it might stand safely. Such a question is of no practical utility, as domes of various sizes and forms have been erected for centuries past, and the question for the architect is,—given a dome of certain form and size, what are the conditions which must obtain between it and the wall of the building it is intended to cover, in order that the whole structure may be in a condition of stability?

The object, therefore, of this paper is to find a solution to the following problem:—

Given a spherical dome, built of stone or brick, of any radius and thickness, and standing on a "drum" or walls of any height; to find the thrust of the dome on the "drum," and the thickness that must be given to the walls in order to insure the stability of the structure.

I take the case of the dome having a spherical section, as being the form most commonly used; but the same method of investigation will apply to domes of any form.

In this investigation I shall consider the dome as made up of a large number of arched ribs, of which the bases resting on the top of the "drum" subtend a small angle (ϕ) at the centre, and the vertices have no thickness; each rib having the form of a wedge cut out of the spherical shell by two planes intersecting in a vertical line through the centre, and making the small angle ϕ with each other. I shall then consider that the two wedges thus formed on opposite sides of the dome, thrust against each other at the vertex, as in the case of an ordinary semicircular arch, and by this means keep each other in equilibrium. Of course no arch of this form if built alone could stand for a moment, as it would give way laterally; but in the dome this is prevented by the parts on each side of the rib under consideration.

This method is adopted by Venturoli in treating on the equilibrium of domes.

Fig 1, plate 4, represents a rib of the form above described cut out of the dome, and the corresponding part of the "drum" which sustains it. The arch will have a tendency to fall in at the crown C D, and open outwards at the haunches E F, causing the joint C D to open at D, and that at E F to open at F.

We may therefore suppose that N, the thrust of the corresponding rib on the opposite side, acts at C.

We shall now find the effect of the force N upon a given joint E F. Let P be the weight of the portion of rib above E F; x the perpendicular distance from E of a vertical from the centre of gravity of P; y the vertical distance of C N from E. Then the moments of these forces about E are Px and Ny; and in order that there may be equilibrium, we must have N equal to the greatest value of $P \cdot \frac{x}{y}$. We have therefore to express $P\frac{x}{y}$ in terms of θ, the angle which F E makes with the vertical, and find what value of θ makes $P\frac{x}{y}$ a maximum.

Let r be the internal and R the external radius of the sphere, δ the weight of a cubic foot of the material of which it is composed. Then the volume of any portion of the rib ($p\,q\,r\,s$ C D) is

$$V = \iiint r^2 \sin\theta \cdot d\theta \cdot dr \cdot d\phi \; ; \quad . \quad . \quad . \quad \text{(A)}$$

so that $P = \delta \cdot V = \delta \cdot \phi\,(1-\cos\theta)\frac{R^3-r^3}{3}$, taking limits from $\theta=0$

If g is the centre of gravity of the portion of the rib whose weight is P, and G its projection on OA; then if $OG=z$, we find the value of z from

$$z = \frac{\iiint r^3 \cdot \sin^2\theta \cdot \cos\phi \cdot dr \cdot d\theta \cdot d\phi}{\iiint r^2 \cdot \sin\theta \cdot dr \cdot d\theta \cdot d\phi}$$

$$= \frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi} \cdot \frac{\iint r^3\,\frac{1}{2}\,(1-\cos 2\theta)\,dr \cdot d\theta}{\iint r^2 \cdot \sin\theta \cdot dr \cdot d\theta} \quad . \quad . \quad \text{(B)}$$

And since ϕ is small, we may consider $\frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}=1$, nearly; in which case

$$z = \frac{3}{8}\,\frac{R^4-r^4}{R^3-r^3}\,\frac{\theta-\frac{1}{2}\sin 2\theta}{1-\cos\theta}$$

Now x is the perpendicular distance of gG from E; therefore

$$x = r\sin\theta - z.$$

Therefore

$$Px = \delta \cdot \phi\,(1-\cos\theta)\,\frac{R^3-r^3}{3} \times \left\{ r\sin\theta - \frac{3}{8}\,\frac{R^4-r^4}{R^3-r^3}\,\frac{\theta-\frac{1}{2}\sin 2\theta}{1-\cos\theta} \right\},$$

or

$$Px = \frac{\delta \cdot \phi}{24}\left\{ 8r\,(R^3-r^3)\sin\theta\,(1-\cos\theta) - 3\,(R^4-r^4)\,(\theta-\tfrac{1}{2}\sin 2\theta) \right\},$$

And since $y = R - r\cos\theta$, and $N = P\frac{x}{y}$,

$$N = \frac{\delta \cdot \phi}{24}\,\frac{8r\,(R^3-r^3)\sin\theta\,(1-\cos\theta) - 3\,(R^4-r^4)\,(\theta-\frac{1}{2}\sin 2\theta)}{R - r\cos\theta}.$$

I will take ϕ as the circular measure of an angle of 2°,

or

$$\phi = \frac{\pi}{90} = {\cdot}0349066,$$

in which case we have

$$N = {\cdot}001454\,\delta\,\frac{8r\,(R^3-r^3)\sin\theta\,(1-\cos\theta) - 3\,(R^4-r^4)\,(\theta-\frac{1}{2}\sin 2\theta)}{R - r\cos\theta}$$

$$= {\cdot}001454\,\delta \times N', \text{ say.}$$

We have now to find what value of θ will make N' a maximum, and as the ordinary rules for maxima will not apply to this expression, we must find it by calculating N' for different values of θ. I have done this for the case when $r=10$, $R=11$, and find that N is greatest when $\theta=70°$; thus

$\theta=69°$ . . . . . $N'=506{\cdot}034$
$\theta=70°$ . . . . . $N'=506{\cdot}241$
$\theta=71°$ . . . . . $N'=506{\cdot}018$.

We have now to substitute in N the values of θ, $\sin\theta$, &c., when the angle is 70°, or when

$$\theta = 35\,\frac{\pi}{90} = 1{\cdot}22173,\ \sin\theta = {\cdot}93969,\ \cos\theta = {\cdot}34202,\ \sin 2\theta = {\cdot}64279.$$

This reduces the expression for N to

$$N = \delta\,\frac{{\cdot}007192\,r\,(R^3-r^3) - {\cdot}0039273\,(R^4-r^4)}{R - {\cdot}34202\,r} \quad . \quad . \quad (1)$$

We can now transpose N and P to the point E; and in order to find the thickness of the pier, we proceed to take their moments, together with those of the part of the rib below E F, and the pier itself, about the outer bottom edge S of the pier.

If we call $yS=b$, the perpendicular distance from S of the direction of N, then

$$b = H + r \cdot \cos\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where H is the height R S of the pier or "drum."

Let $Se=a$, the distance from S of a vertical dropped from E; F=the weight of the portion of the rib below E F; c=the perpendicular distance from S of a vertical from the centre of gravity of the lower portion of the rib whose weight is F; Q=the