$$A B = A C \frac{\sin}{\sin(90^\circ - \frac{\phi}{2})} = a \frac{\sin}{\cos\frac{\phi}{2}}$$

$$\therefore A E = \frac{a \sin\phi}{\cos\frac{\phi}{2}} \cdot \frac{\cos(\frac{\phi}{2} + \theta}{\sin\phi} = a \frac{\cos(\frac{\phi}{2} \pm \theta)}{\cos\frac{\phi}{2}}$$

$$= a \cos\theta \pm \sin\theta \,.\, \tan\frac{\phi}{2}$$

Where ϕ is very small, *i. e.*, when B is indefinitely close to A, $A E = a \cos\theta$, which shows that C E A is a right angle.

To calculate the disc of confusion of focus, we will take the absolute size of the grating, taking P and Q as the external edges of the grating.

In this case, $\tan\phi = \frac{1}{48}$, since $A C = a = 12$ feet and $A B = 3$ inches $= \frac{1}{2}$ width of grating.

Suppose $\theta = 30^\circ$. Then $a \cos\theta = 144 \times \cos 30^\circ = 121{\cdot}7$ inches.

$$A \sin\theta \cos\frac{\phi}{2} = 144 \sin 30 \cos 36' = {\cdot}754 \text{ inch.}$$

A further calculation will show that the disc of confusion or breadth of a point would be

$$= {\cdot}754 \tan\frac{\phi}{2} = {\cdot}015.$$

The confusion of this disc would be almost inappreciable at the edges; in fact, we may take it to begin to be appreciable at $\frac{1}{4}$ that diameter. The breadth of a point may therefore be taken at about $\frac{4}{1000}$ of an inch, which is well within the limits admitted to give a sharp focus, and is better than that which can be got from a lens under similar circumstances.

The same problem may be solved geometrically. Using the same notation as before, and assuming B to be very close to A, it follows that A B is very small compared with C D or C K; and it will be seen that D K = A B, taking A B as parallel to C D; therefore, for all intents, C D may be taken = C K.

Now the triangle A H C and B H E are similar, as are the triangles B H A and H C K.

$$\therefore \frac{BH}{HC} = \frac{AB}{CD} = \text{or } \frac{BH + HC}{HC} = \frac{AB + CD}{CD}$$

$$\text{But } \frac{BH + HC}{HC} = \frac{BC}{HC} = \frac{AC}{HC} \text{ and } \frac{AB + CD}{CD} = \frac{CK}{CD} = \frac{CD}{CD} = 1$$

$$\text{(i.) } \therefore \frac{AC}{HC} = 1 \text{ or } AC = HC;$$

that is, H is very close to A and B.

Again,

$$\frac{HE}{HC} = \frac{BH}{AH}.$$

But

$$\frac{HE}{HC} = \frac{AE}{AC},$$

since H if very close to A, A B being small.

$$\text{(ii.) } \therefore \frac{AE}{AC} = \frac{BH}{AH},$$

now B H and A H are both small; and

$$\therefore \frac{BH}{AH}$$

might be very large, and therefore cannot be reflecte'.

Now,

$$\frac{AH}{HK} = \frac{BH}{HC} \therefore AH = \frac{HK \times BH}{HC}$$

Substituting this value of A H in (ii.)—

$$\text{(iii.) } \frac{AE}{AC} = \frac{BH \times HC}{HK \times BH} = \frac{HC}{HK}$$

Now both H C and H K are large quantities.

$$\therefore \frac{HC}{HK} = \frac{AC}{AK}$$

since H is indefinitely near A.

Substituting in iii. we get—

$$AE \times AK = AC^2.$$

If a circle be described about C E K, it follows, since this relation holds good, that A C must be a tangent to the circle; and as, by hypothesis, C K is at right angles to A C, therefore the arc C E K is a semicircle; since arc C E K is a semicircle C E K must be a right angle. That is, the focus for the rays is found by letting fall a perpendicular from the centre of curvature on to the reflected ray; or if the focus of the reflected rays be at the centre of curvature, the focus for the incident ray must be found in the same way.

This last is what Professor Rowland carries out in practice the reflected ray is also reflected towards the centre of the sphere of which G is a segment; the distance between the grating and the plate or focussing screen remains unchanged, and the distance between the slit and the grating is altered. To effect this he has two bars at right angles to one another, with a third bar

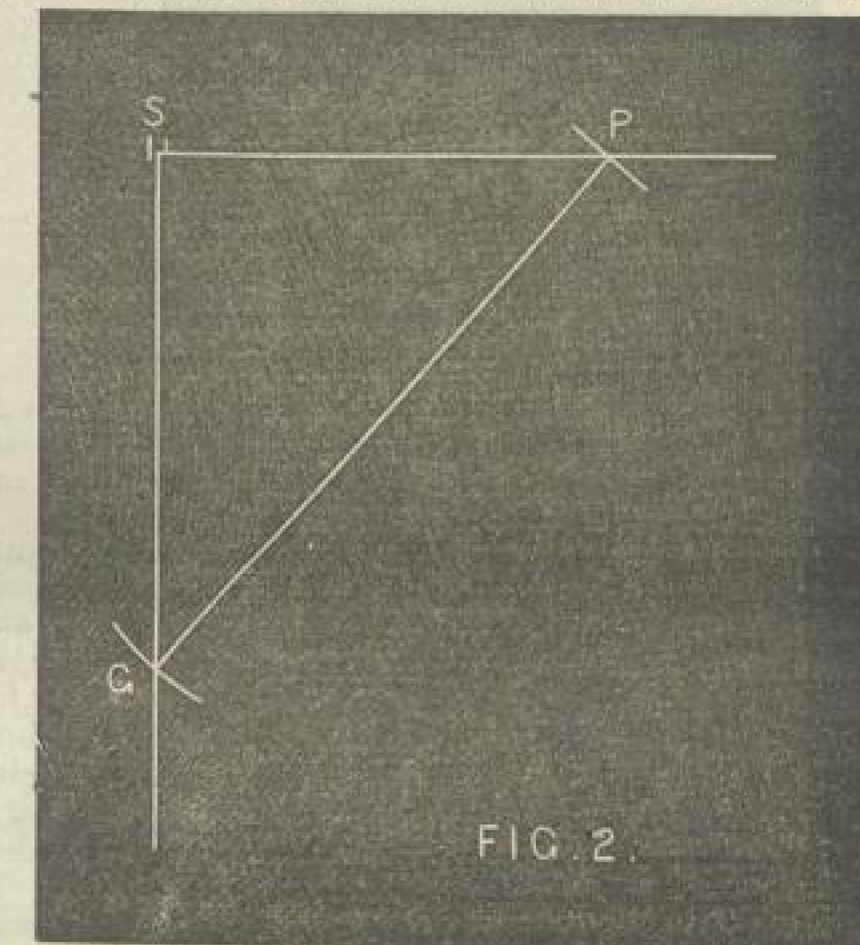

FIG. 2.

sliding along them. This bar carries the grating G at one end, and the plate P at the other, the centre of the plate P occupying the centre of the circle of which the grating G is an arc. The slit S is fixed. It will be seen that this fulfils the requirements of the theorem just given. By keeping the centre of the grating at P a true normal spectrum is always thrown, and, however the angle S G P is altered so as to get different parts of the spectrum, the scale of the photographs remains unchanged, since the distance from G to P is fixed.

I would also ask you to remark, that as the angle is increased so is the slit placed nearer to the grating, which means that a larger cone of light, and consequently greater brilliancy of spectrum, is given than would otherwise be obtained; this is, however, at the expense of the fineness of the lines, since the breadth of a line is the disc of confusion of a point + breadth of slit $\times \frac{\text{distance of plate from grating}}{\text{distance of slit from grating}}$. This is, however, more than compensated for by the fact that if you largely increase the angle S G P, Fig. 2, you work in higher orders of the spectrum, which give increased dispersion, and do not get a proportionate shortening of the distance of the slit from the grating. Thus we have already taken an angle of 30° as an example, and found that the total shortening of the slit is 124·7 inches.

If we take 60°, which will give us the same rays of the 2nd order we find that the focal distance is reduced 144. cos. 60 = 72 inches. In this last case the image of the line will be $\frac{124{\cdot}7}{72}$ broader, for which the dispersion is doubled; there will also be a slight increase in the disc of confusion.

The brightness is increased by $\left(\frac{124{\cdot}7}{72}\right)^2$, or nearly 2·8 times that which would be the case supposing the focal distance of the slit remained at 72 inches. A certain diminution in this amount must be made, owing to the height of the slit being magnified as well as the breadth; but, owing to the grating being spherical, the edges of the spectrum are less intense than the central portion, which is the part of importance, most of the light being collected there.

Another property I would call your attention to. C and E are conjugate foci, as are C and E'.

$\therefore$ E and E' are also conjugate foci.

If, therefore, the slit and the photographic plate are pivoted about O, with arms of length O E, they may occupy any position and still will remain in focus. The photographic plate