Zaunick V/17. Nr. 4

Aufgaben mit Lösungen.

von E. W. von Tschirnhaus' Hand.

Anscheinend in England geschrieben: Papier mit dem Wasserzeichen außerdem gelegentlich das englische Zeichen ∴ (therefore)

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V

44 Seiten

starker Papierzerfall, mikrofilmen!

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Habeo numerum si illius 3tiam partem in tertiam sui 4tam partem multiplico, provenit ejusdem quadruplum:

$3 \propto a$
$4 \propto b$

$\frac{x}{a} - \frac{xx}{a}$

$\frac{x^3}{aa} \propto \frac{bx}{1}$

$x^3 \propto aabx$

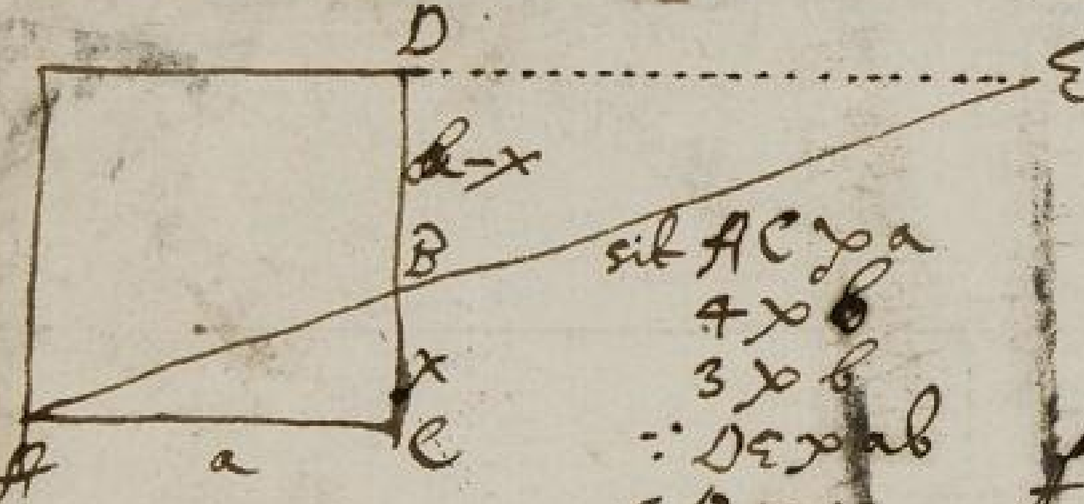
$xx \propto aab$

$x \propto \sqrt{aab}$

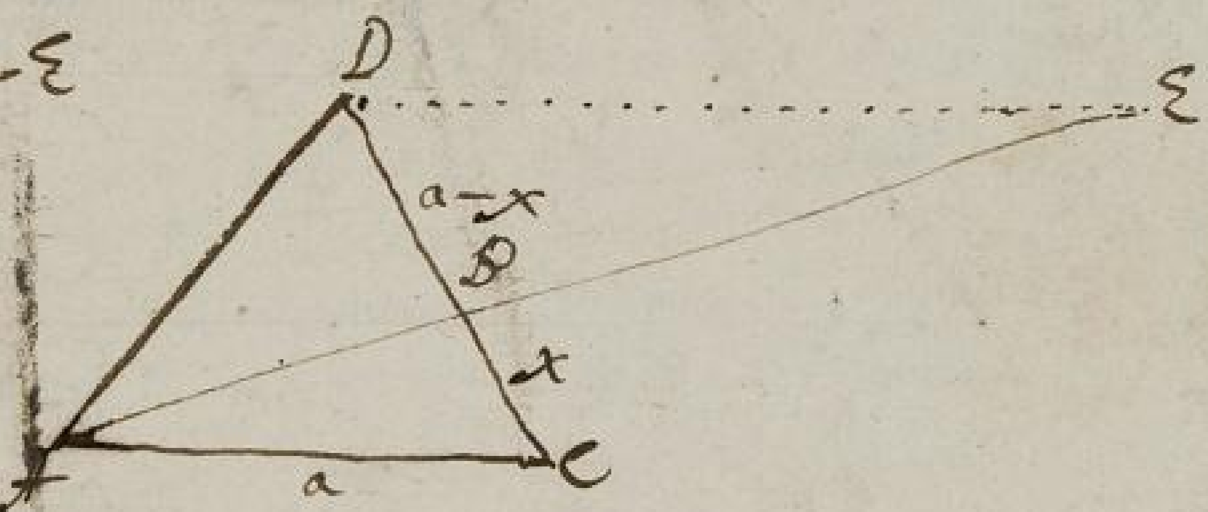
$\frac{8}{3} | 2 \quad \frac{36}{33} | 12 \quad \frac{12}{2}$ / $\frac{24}{9} | 4$

43

Dato Quadrato et Triangulo aequilatero; et producta DE parallela AC et indefinita; ex latere DC abscindere certam partem DC ita ut producta linea AE ipsa A incipiens, et transiens in B et in E terminans afferat inde lineam DE $\propto$ Ambitui dati Quadrati et Δ aequilateri:



sit AC $\propto$ a
4 $\propto$ b
3 $\propto$ b
∴ DE $\propto$ ab
CB $\propto$ x
∴ BD $\propto$ a−x



Propter Δ sim: ABC et EBD erit ut

AC a — π — BC x — π — ED ab π BD a−x

$ab - ax \propto abx$

$bx + x \propto a$

$x \propto \frac{a}{b+x}$

Hinc patet in Quadrato CB debere esse 3tiam totius CD partem in Δlo vero 4tam totius CD.

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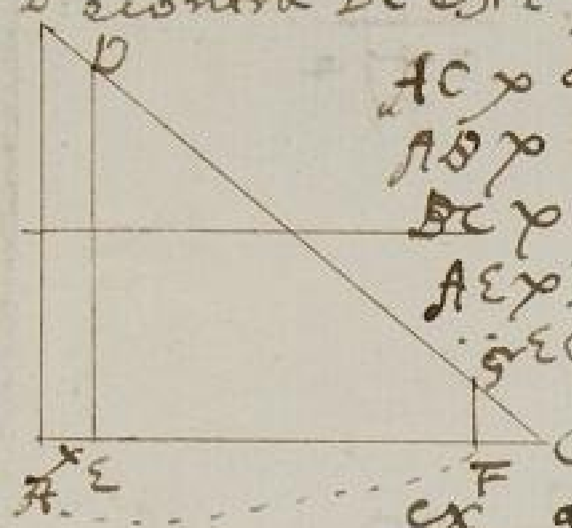
Dati Trianguli ABC latus AC ita dividere in C ut ducta ED perpendiculari ~~in AC~~ basi in D terminante segmentum BD et EC summa aequalis sit differentiae segmentorum DC et AE: et ut e contra DC et AE summa $\propto$ sit differentia BD et EC.



$AC \propto a$ $\quad a \;\pi\; c \;\pi\; a - x \;\pi\; \frac{ac - cx}{a} \propto DC$:

$AB \propto b$

$BC \propto c$ $\quad c - \frac{ac - cx}{a}$

$AE \propto x$

$EC \propto a - x$ $\quad \frac{cx}{a} \propto BD$: jam itaque quoad

(1)

pro: 2. $\quad x$

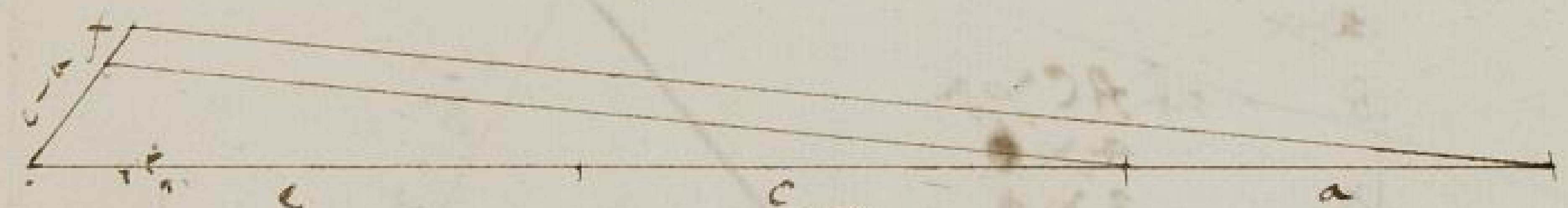
$$\frac{cx}{a} \quad \frac{a - x}{1} \qquad \frac{ac - cx}{a} \quad \frac{x}{1} \qquad \text{NB } 2cx + aa \propto ac$$

$$\frac{cx \mp aa - ax}{a} \propto \frac{ac \mp cx - ax}{a}$$

$$2cx \propto ac - aa$$

$$x \propto \frac{ac - aa}{2c} \text{ hinc } 2c \;\pi\; \cancel{a} \;\pi\; c - a \;\pi\; x$$

A



(2)

$$\frac{cx}{a} \quad \frac{a - x}{1} \qquad \frac{ac - cx}{a} \quad \frac{x}{1} \qquad \text{NB } 2cx \propto ac + aa$$

$$\frac{cx \mp aa + ax}{a} \propto \frac{ac \mp cx + ax}{a}$$

$$2cx \propto ac + aa$$

$$x \propto \frac{ac + aa}{2c} \text{ hinc } 2c \;\pi\; a \;\pi\; \cancel{a} + c \;\pi\; x$$

B

2

Idem aliter

Sit $CE \propto x$ $\therefore AE \propto a-x$

$a \;\pi\; c \;\pi\; x \;\pi\; \frac{cx}{a} \propto CD$ $\quad c - \frac{cx}{a}$

$\frac{ac-cx}{a} \propto BD$

Hinc pro x erit

$$\frac{ac-cx}{a} \times \frac{x}{1} \qquad \frac{cx}{a}\;\frac{a-x}{1}$$

$$\frac{ac \mp cx + ax}{a} \propto \frac{cx \pm aa + ax}{x}$$

$2cx \propto aa + ac$

$x \propto \frac{ac+aa}{2c}$ C

$$\frac{ac-cx}{a} \times \frac{x}{1} \qquad \frac{cx}{a}\;\frac{a-x}{1}$$

$$\frac{ac \mp cx - ax}{a} \propto \frac{cx \mp aa - ax}{x}$$

$2cx \propto ac - aa$

$x \propto \frac{ac-aa}{2c}$ D

Hinc patet si prima linea AE sit inventa; ~~secunda quae secundo quaesitae~~, hacque ex C in F translata; ducta perpendiculari FG lineas BC et AC in talia segmenta divisum iri quae secundo quaesito satisfaciant. Aequali enim ~~Aidem~~ ab utrobique lineis AF et CE (juxta aequali BDC) sublato (nempe EF) remanent AE $\propto$ FC:

Et sic econtra FC primo inventa habebis lineam AE quae ~~primo~~ quaesito satisfaciat.

Idem adhuc aliter.

Sit $BD \propto x$ $\therefore DC \propto c-x$

$c \;\pi\; a \;\pi\; c-x \;\pi\; \frac{ac-ax}{c} \propto CE$

$a - \frac{ac-ax}{c}$

$\frac{\cancel{ac}\;ax}{c} \propto AE$ Hinc pro x erit:

$$x \;\frac{ac-ax}{c} \qquad \frac{c-x}{1}\;\frac{ax}{c}$$

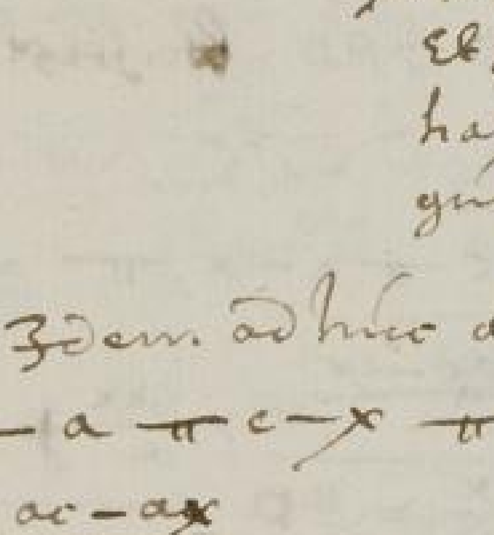
$$\frac{cx \mp ac - ax}{c} \propto \frac{cc \mp cx - ax}{c}$$

$2cx \propto cc - ac$

$2x \propto c - a$

$x \propto \frac{c-a}{2}$

Nb. $2x + a \propto c$



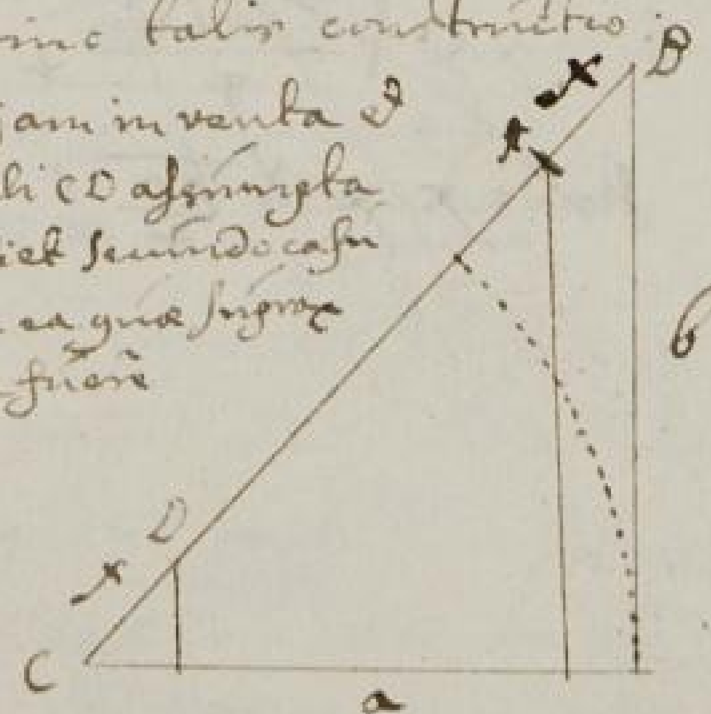
Proba

$\frac{1}{1}\;\frac{12}{5}$ | $4\frac{3}{5}$

$\frac{17}{5}$ f. 7 | $\frac{17}{5}$.

Hinc talis constructio.

AB jam inventa et aequali CD assumpta satisfiet secundo casu juxta ea quae supra dicta fuere



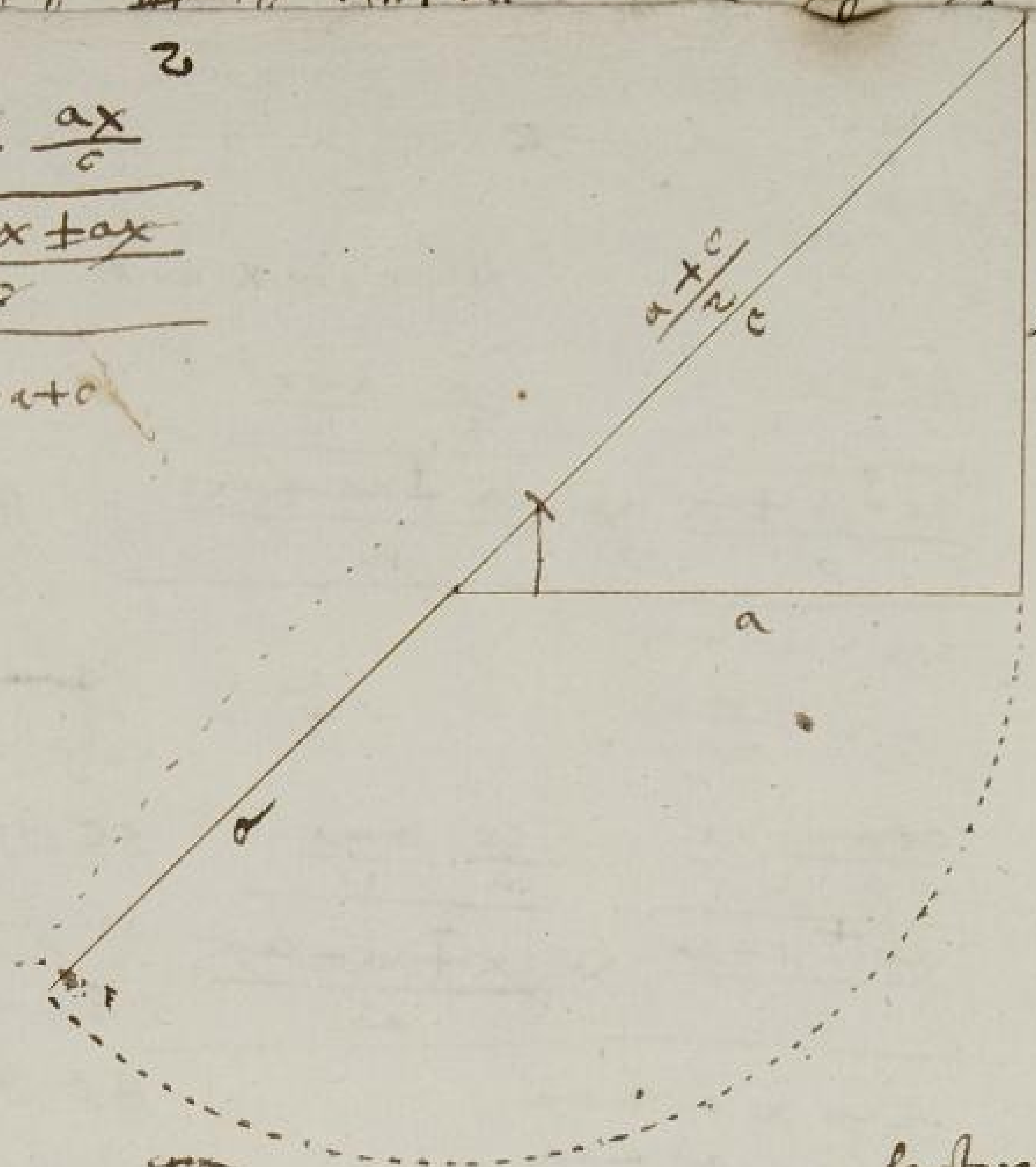
$\frac{x}{1} + \frac{ac-ax}{c}$ $\frac{c-x}{1}$ $\frac{ax}{c}$

$\frac{cx \pm ac \pm ax}{c} \propto \frac{cc \pm cx \pm ax}{c}$

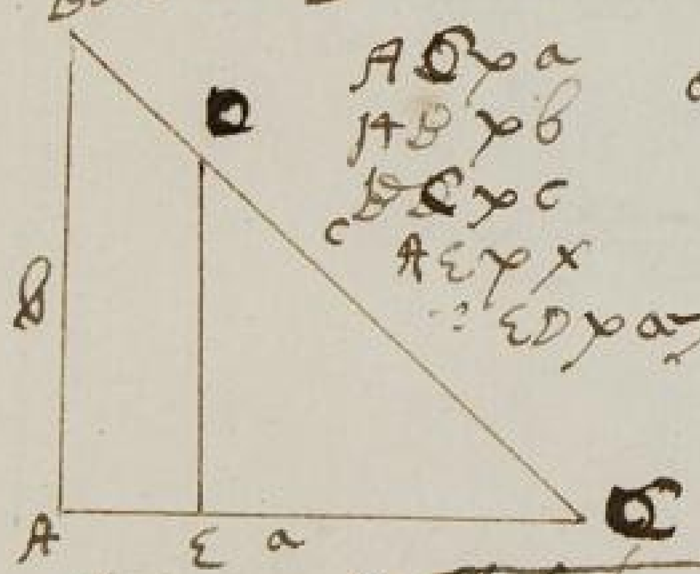
$2cx \propto ac + cc$ N.B.: $2x \propto a+c$

$2x \propto a+c$

$x \propto \frac{a+c}{2}$



Dati Trianguli Rect: ABC latus AD ita dividere in E ut quadrata hinc EC perpendicularis lateris BD et AD efficiunt segmentis BC et CD item BAE et ED) ubi sunt BCAE et CDE; sint aequalia □ BD



AD $\propto$ a
AB $\propto$ b
DC $\propto$ c
AE $\propto$ x
ED $\propto$ a−x

$a \ \pi \ c \ \pi \ a-x \ \pi \ \frac{ac-cx}{a} \propto CD$

$c - \frac{ac-cx}{a}$ / $\frac{cx}{a} \propto BD$. $\frac{cxx}{a} + \frac{aac-2acx+cxx}{a}$

$\frac{aac-2acx+2cxx}{a} \propto \frac{cc}{1}$

$aac - 2acx + 2cxx \propto acc$

$2cxx \propto 2acx + acc - aac$

$xx \propto ax + \frac{acc-aac}{2c}$

$x \propto \frac{a}{2} + \sqrt{\frac{aa}{4} \ \frac{acc-aac}{2c}}$

$\frac{2acc - aac}{2c}$

$x \propto \frac{a}{2} + \sqrt{\frac{2ac-aa}{4}}$ #

Si: $x \propto \sqrt{\frac{2ac \pm aa}{4}} + \frac{a}{2}$ Si: $\frac{aa}{4}$

~~Unde $x \propto \sqrt{\frac{ac}{4}}$~~

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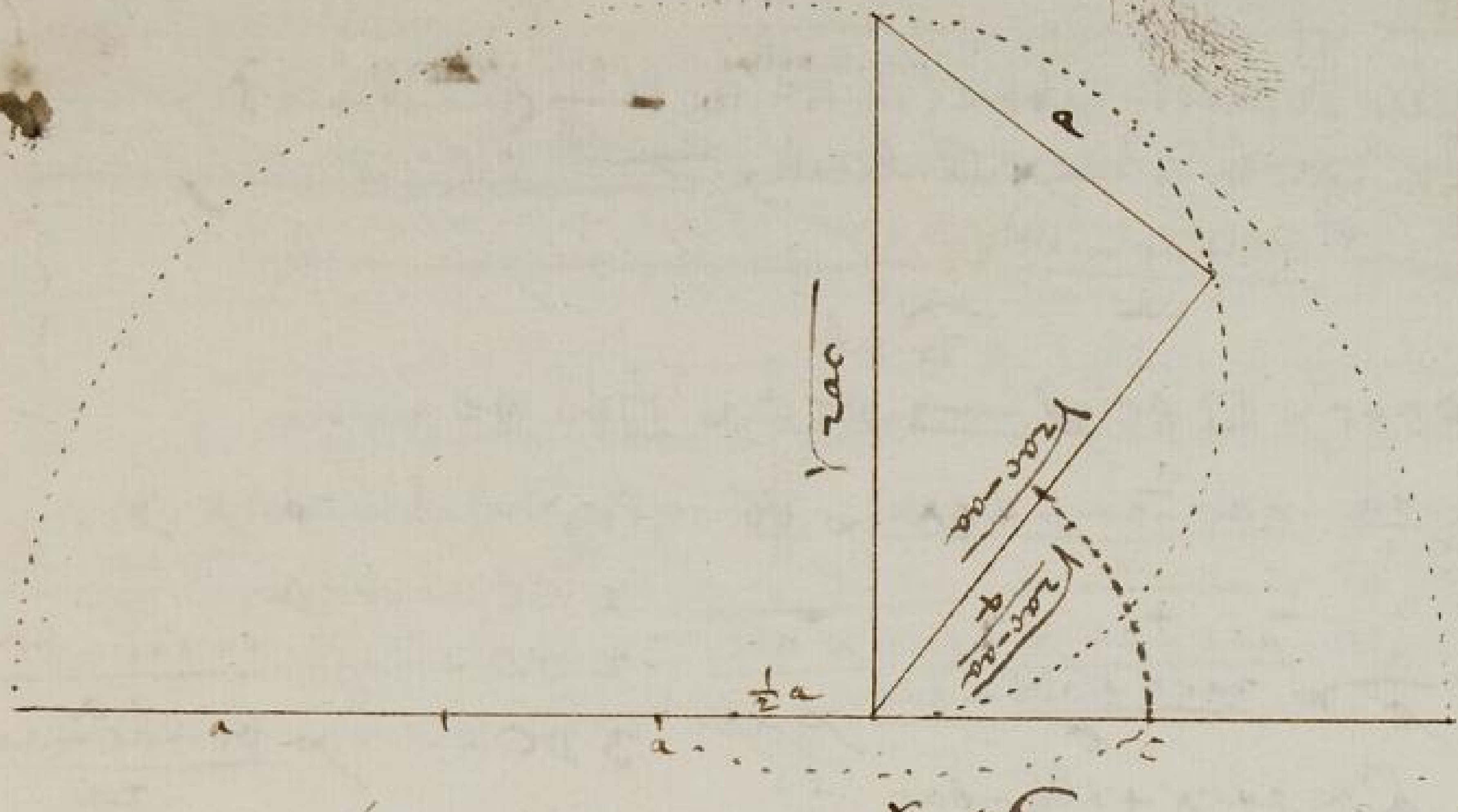
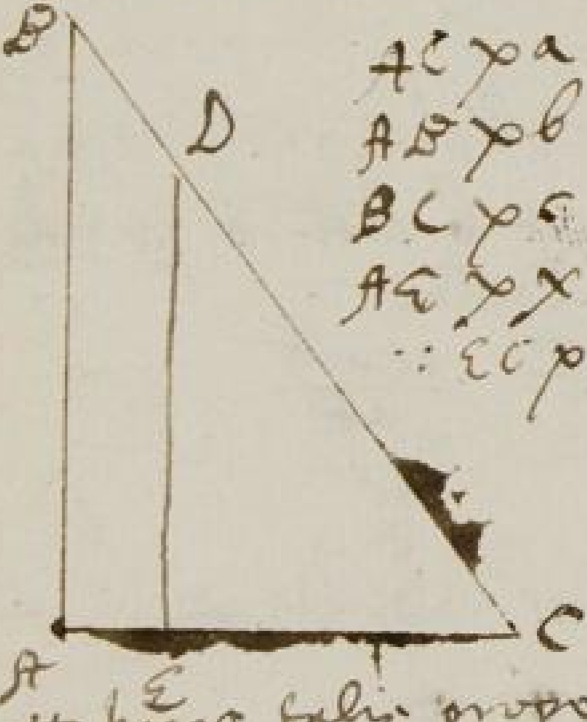


IB:

x 4.6

Dati Trianguli ABC latera AC et BC et erecta ex AC perpendicularem ED ita dividere ut ☐ ~~...~~ segmentis DC et AE sit $\propto$ ☐to AB ☐ autem ex DC in EC $\propto$ ☐ AC. vel econtra; sit itaque primo



AC $\propto$ a
AB $\propto$ b
BC $\propto$ c
AE $\propto$ x
∴ EC $\propto$ a − x

$a \pi c \pi a - x \pi \frac{ac - cx}{a} \propto DC$

$c - \frac{ac - cx}{a}$

$\frac{cx}{a} \propto BD$

$\frac{cxx}{a} \propto \frac{bb}{x}\ \frac{bb}{1}$ $\quad \frac{aac - 2acx + cxx}{a} \propto \frac{aa}{1}$

$\frac{cxx}{a} \propto \frac{bb}{1}$ $\quad aac - 2acx + cxx \propto a^3$

$cxx \propto a^3 + 2acx - aac$

$cxx \propto abb$

$xx \propto \frac{abb}{c} \propto \frac{a^3 + 2acx - aac}{c} \propto xx$

$bb \propto aa + 2cx - ac$

$2cx \propto bb - aa + ac$

$x \propto \frac{bb + ac - aa}{2c}$ #

hinc talis proportio.

$2c \pi \sqrt{bb+ac-aa} \pi \sqrt{bb+ac-aa} \pi x.$

NB. si EC $\propto$ x sumpsisses proventuras fuisset aequatio in qua $x \propto \frac{aa + ac - bb}{2c}$ jam inventa id quod statim ex hoc prima aequatione patet cum enim EC sit $\propto$ a − x ab a subtrahendo $x \propto \frac{bb + ac - aa}{2c}$ ~~jam inventa~~ proveniet modo indigitata aequatio. 2 si x $\propto$ sumpsisses BD ut cognoscas quanam ~~...~~ futura sit aequatio; accipe ~~...~~ quantitatem $BD \propto \frac{cx}{a}$ et vertendo x in eadem jam inventa quantitate x; invenies desideratum nempe $x \propto \frac{bb + ac - aa}{2a}$; Tandem

5

si scire desideras; quidnam assumpto DC ∞ x apparitura sit aequatio; et hoc ipsum quidem ex hac jam semel inventa aequatione nullo alio adhibito labore: restituta (hoc est substituta) in quantitate DC ∞ $\frac{ac-cx}{a}$ incognitae jam inventa quantitate x; ~~simul tabilia videbis, sive~~ ~~tunc porit~~ $\frac{aa+ac-bb}{2a}$ ~~id quod velim maxime notes~~.

(2) sit

▭ BDAE ∞ □ AC et ▭ DCE ∞ □ to AB nimi;

$\frac{cxx}{a} = \frac{aa}{1}$ $\frac{aac - 2acx + cxx}{a} = \frac{bb}{1}$ itaque si assumatur x ∞

$cxx = a^3$ $aac \mp 2acx \pm cxx = abb$ 1 EC provenit ∞ $\frac{bb+ac-aa}{2c}$

$xx = \frac{a^3}{c} = \frac{2acx + abb - aac}{c} = xx$ 2 BD --- ∞ $\frac{aa+ac-bb}{2a}$

3 DC ---- ∞ $\frac{bb+ac-aa}{2a}$

$a^3 = 2acx + abb - aac$

$aa = 2cx + bb - ac$

$2cx = aa + ac - bb$

$x = \frac{aa+ac-bb}{2c}$

47.

1 pars Quaestionis: Habeo 3 numeros, si ex primi et secundi producto extrahatur radicem quadratam remanet a si ex secundi ex tertii remanet b et tandem ex tertii et primi c:

$\sqrt{xy} = a$	$\sqrt{yz} = b$	$\sqrt{zx} = c$	Proba $\frac{aabc}{bc}$	$\frac{bbac}{ac}$	$\frac{ccab}{ab}$
$xy = aa$	$yz = bb$	$zx = cc$	$\sqrt{aa}$	$\sqrt{bb}$	$\sqrt{cc}$
$y = \frac{aa}{x}$	$\frac{aaz}{x} = \frac{bb}{1}$	$\frac{bbxx}{aa} = \frac{cc}{1}$	a.	b.	c.
$y = \frac{ab}{c}$	$aaz = bbx$	$bbxx = aacc$			
	$z = \frac{bbx}{aa}$	$xx = \frac{aacc}{bb}$			
	$z = \frac{bc}{a}$	$x = \frac{ac}{b}$			

Numeri hinc deducti #

~~4. 1. 16 9. 1.~~

~~12 3 48~~

16 4 64

~~32 8 128~~

4.	1	16	9.	1.	81	16.	1.	256
8	2	32	18.	2.	162	32	2	512
12	3	48	27	3	243	48	3	768
16.	4.	64.	36	4	324	64	4	
20.	5.	80	45	5	405	80	5	
24.	6.	96	54	6	486	96	6	

Hinc ad infinitos tales numeros inveniendos sequens regula formari poterit, assume quadratum aliquem numerum eius et dextris adjunge unitatem, huicque quadrati eiusdem quadratum; et habebis desiderates hos si duplicaveris, triplicaveris etc. infiniti tunc quaesito satisfacientes surgent. (Cent 237)

2 Assumas tres quoscunque numeros, quibus sic procedes (& proportione): dicendo ut primus ad secundum sic tertius ad 4tum (2 π 3 π 4. erit 6) quem seorsim reponens, tunc ulterius pergendo dices ut 2us ad tertium sic primus ad quartum (3 π 4 π 2 erit $\frac{8}{3}$) priori invento a sinistro adjunges, Tandem ultimo ut 3tius ad primum sic 2dus ad 4tum (4 π 2 π 3 erit $\frac{3}{2}$) et hunc intermedio prius inventorum collocabis, hisce sic positis, dico productum primi et secundi aequari quadrato assumpti primi numeri; productum vero secundi et tertij quadrato secundi et denique tertij et primi quadrato ultimi.

3 Hinc constat ratio inveniendi 3 numeros, quorum 1 et 2 productum sit Quadratus numerus, 2di et tertij, denique et tertij et primi

(2)

Habeo numerum 21 ~~quem~~ in tres ejusmodi partes dividendum, ut radix producti primae et secundae partis sit = 6 s: 2 secundae et tertiae = 4 s: d et tandem tertiae et primae = 8 s: d.

x = prima pars; y = secund: ∴ a − x − y

$\sqrt{xy} = b$

$xy = bb$

$y = \frac{bb}{x}$

$\sqrt{ay-xy-yy} = c$

$ay - xy - yy = bb$

$\frac{abb}{x} - bb - \frac{b^4}{xx} = cc$

$\frac{abbxx - bbx^3 - b^4x}{x^3} = cc$

$abbx - bbxx - b^4 = ccxx$

$ccxx + bbxx = abbx - b^4$

$xx = \frac{abbx - b^4}{bb+cc}$

$\sqrt{ax - xx - xy} = d$

$ax - xx - xy = dd$

$ax - xx - bb = dd$

$xx = \frac{ax - bb - dd}{1} = \frac{abbx - b^4}{bb+cc}$

$\cancel{abbx - b^4 = bbdd + abbx}$

$bbax - b^4 = bbdd + ccax - bbcc - ccdd + abbx - b^4$

$ccax = bbdd + bbcc + ccdd$

$x = \frac{bbdd + bbcc + ccdd}{cca}$

s: $x = \frac{bbdd}{cca} + \frac{bb}{a} + \frac{dd}{a}$

3

Habeo numerum ~~quem~~ in tres ejusmodi partes divisum, quarum radix producti primae et secundae sit = 2; secundae et tertiae 4 et tandem tertiae et primae = 8.

Sit numerus = x cujus prima pars = y secunda = z ∴ Tertia = x − y − z sit denique 2 = a 4 = b 8 = c.

$\sqrt{\cancel{yz}} \propto a$ $\qquad$ $\sqrt{xz - yz - zz} \propto b$ $\qquad$ $\sqrt{xy - yy - yz} \propto c$

$yz \propto aa$ $\qquad$ $xz - yz - zz \propto bb$ $\qquad$ $xy - yy - yz \propto cc$

$z \propto \frac{aa}{y}$ $\qquad$ $\frac{aax}{y} - \frac{aa}{1} - \frac{a^4}{yy} \propto bb$ $\qquad$ $\frac{bby^3 + aay^3 + a^4y}{aay}$

$z \propto \frac{ab}{c}$

$$\frac{aax - aay}{y} \quad \frac{a^4}{yy}$$

$$\frac{aaxyy - aay^3 - a^4y}{y^3}$$

$$\frac{aaxy - aayy - a^4}{yy} \propto \frac{bb}{1}$$

$$aaxy - aayy - a^4 \propto bbyy$$

$$x \propto \frac{bbyy + aayy + a^4}{aay}$$

$$\frac{bbyy + aayy + a^4}{aa} - yy - aa \propto cc$$

$$\frac{bbyy + aayy + a^4}{aa} \propto \frac{cc + yy + aa}{1}$$

$$bbyy + \cancel{aayy} + \cancel{a^4} \propto aacc + \cancel{aayy} + \cancel{a^4}$$

$$yy \propto \frac{aacc}{bb}$$

$$y \propto \frac{ac}{b}$$

Hoc est $x \propto \frac{bc}{a} + \frac{ac}{b} + \underline{ab}$ $\qquad$ Hinc $x - y - z$ erit $\propto \frac{bc}{a}$:

Hinc patet qua ratione hoc ultimum maximo cum compendio ex quaestionis Item prima parte possit inveniri: qd notandum.

48:

Habeo 3 numeros; primi et Secundi summa $\propto 6$, secundi et Tertij $\propto 10$, Tertij denique et primi $\propto 8$: $\quad$ sit $6 \propto a$ $\; 10 \propto b$ $\; 8 \propto c$

Prima solutio

$x + y \propto a$ $\qquad$ $a - x + z \propto b$ $\qquad$ $b + x - a$

$y \propto a - x$ $\qquad$ $z \propto b + x - a$ $\qquad$ $+x$

$$b + 2x - a \propto c$$

$$x \propto \frac{a + c - b}{2}$$

Secunda solutio

NB. $x + y \propto a$

$y + z \propto b$

$z + x \propto c$

$2x + 2y + 2z \propto a + b + c$

$x + y + z \propto \frac{a + b + c}{2}$

$\cancel{x} + y + z \propto b$

$x \propto \frac{b + c - a}{2}$ $\qquad$ $y \propto \frac{a + c - b}{2}$ $\qquad$ $z \propto \frac{a + b - c}{2}$

Tertia solutio

$x+y=a$ $\quad y+z=b$ $\quad z+x=c$

$y+z=b$ $\quad z+x=c$ $\quad x+y=a$

$2y+x+z=a+b$ $\quad 2z+x+y=b+c$ $\quad 2x+y+z=a+c$

$$y=\frac{a+b-x-z}{2} \qquad 2z+\frac{a+b-x-z}{2}+x=b+c \qquad \frac{a+b-x-z}{2}+\frac{b+2c-a-x}{3}=y+z$$

$$\frac{a+b-x-z}{2}=\frac{b+c-x-2z}{1} \qquad \frac{3a+3b-3x-3z+2b+4c-2a-2x}{6}\ \text{hoc } \bar{e}$$

$$a+b-x-z=2b+2c-2x-4z \qquad \frac{a+5b-5x-3z+4c}{6}$$

$$3z=b+2c-a-x \qquad \frac{a+5b-5x+4c}{6}-\tfrac{1}{2}z=\frac{b+2c-a-x}{6}$$

$$z=\frac{b+2c-a-x}{3} \qquad \frac{2a+4b+2c-4x}{6}\ (2$$

$$\frac{a+2b+c-2x}{3}=x+y \quad \text{addatis } 2x \text{ transferatis in altera parte erit}$$

$$\frac{a+2b+c-2x}{3}=\frac{a+c-2x}{1}$$

$$a+2b+c-2x=3a+3c-6x$$

$$4x=2a+2c-2b$$

$$x=\frac{a+c-b}{2}.$$

Quarta solutio

$x+y=a$ $\quad x+y=a$ $\quad y+z=b$ $\quad z+x=c$

$y+z=b$ $\quad \therefore x=a-y$ $\quad \therefore y=b-z$ $\quad z=c-x$

$z+x=c$

$2x+2y+2z=a+b+c$

$x+y+z=\frac{a+b+c}{2}$ itaq.

$$\therefore x=\frac{a+b+c}{2}-y-z=a-y=x \qquad \therefore y=\frac{a+b+c}{2}-x-z=b-z=y$$

$$\frac{a+b+c}{2}=\frac{a+z}{1} \qquad \frac{a+b+c}{2}=\frac{b+x}{1}$$

$$a+b+c=2a+2z \qquad a+b+c=2b+2x$$

$$z=\frac{b+c-a}{2} \qquad x=\frac{a+c-b}{2}$$

Quamquam haec ad solutionem propositi sufficiant, poterit tamen ex iisdem etiam inveniri y sequenti ratione:

3

$\therefore z = \frac{a+b+c}{2} - x - y = c - x = z$

$\frac{a+b+c}{2} = \frac{c+y}{1}$

$a+b+c = 2c+2y$

$\therefore y = \frac{a+b-c}{2}$

Quinta Solutio

$x+y=a \quad y+z=b \quad z+x=c$

Ergo

$x = a-y$ | $y = b-z$ | $z = b-y$
$x = c-z$ | $y = a-x$ | $z = c-x$
$x = \frac{a+c-y-z}{2}$ | $y = \frac{b+a-x-z}{2}$ | $z = \frac{b+c-y-x}{2}$

~~itaque x~~ His ita ~~positis~~ tribus diversis modis desiderata acquires

~~$x = \frac{a+c-}{}$~~

1

$x = \frac{a+c-y-z}{2} = \frac{a-y}{1} = x$

$a+c-y-z = 2a-2y$

itaque x

$z = c+y-a = b-y = z$

$y = \frac{a+b-c}{2}$

Hinc z

$y = a+z-c = b-z = y$

$z = \frac{b+c-a}{2}$

2

$y = \frac{b+a-x-z}{2} = \frac{b-z}{1} = y$

$b+a-x-z = 2b-2z$

Hinc x

$z = b-a+x = c-x = z$

$x = \frac{a+c-b}{2}$

Hinc z

$x = a-b+z = c-z = x$

$z = \frac{b+c-a}{2}$

3

$z = \frac{b+c-y-x}{2} = \frac{b-y}{1} = z$

$b+c-y-x = 2b-2y$

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Hinc x $\qquad\qquad$ Hinc z

$y \propto b + x - c \propto a - x \propto y \qquad x \propto c + y - b \propto a - y \propto x$

$x \propto \frac{a+c-b}{2} \qquad y \propto \frac{b+a-c}{2}.$

Sexta solutio.

$$\begin{array}{lll} x+y \propto a & y+z \propto b & z+x \propto c \\ y+z \propto b & z+x \propto c & x+y \propto a \\ \hline 2y+x+z \propto a+b & 2z+x+y \propto b+c & 2x+y+z \propto a+c \end{array}$$

$y \propto \frac{a+b-x-z}{2} \propto \frac{b+c-x-2z}{1} \propto y$

$a+b-x-z \propto 2b+2c-2x-4z$

$x \propto a-y \quad y \propto b-z \quad z \propto b-y$

$x \propto c-z \quad y \propto a-x \quad z \propto c-x$

Hinc x $\qquad\qquad$ Hinc z

$x \propto \frac{b+2c-a-3z}{2} \propto c-z \propto x \qquad z \propto \frac{b+2c-a-x}{3} \propto \frac{c-x}{1} \propto z$

$z \propto \frac{b+c-a}{2} \qquad b+2c-a-x \propto 3c-3x$

$x \propto \frac{a+c-b}{2}.$

Atque ita satisfactum esset proposito, sed iidem duabus adhuc viis ex premissis inveniri potest.

$z \propto \frac{b+c-y-x}{2} \propto \frac{a+c-2x-y}{1} \propto z$

$b+c-y-x \propto 2a+2c-4x-2y$

$b \propto 2a+c-3x-y$

Hinc x $\qquad\qquad$ Hinc z

$x \propto \frac{2a+c-b-y}{3} \propto a-y \propto x \qquad x \propto \frac{2a+c-y-b}{3} \propto \frac{a-y}{1} \propto x$

$x \propto \frac{a+c-b}{2} \qquad 2a+c-y-b \propto 3a-3y$

$y \propto \frac{b+a-c}{2}$

$x \propto \frac{a+c-y-z}{2} \propto \frac{a+b-z-2y}{1} \propto x$

$a+c-y-z \propto 2a+2b-2z-4y$

$c \propto 2b-3y-z+a$

Hinc x $\qquad\qquad$ Hinc z

$z \propto 2b+a-c-3y \propto b-y \propto z \qquad y \propto \frac{2b-z+a-c}{3} \propto \frac{b-z}{1} \propto y$

$y \propto \frac{b+a-c}{2} \qquad 2b-z+a-c \propto 3b-3z$

$z \propto \frac{b+c-a}{2}$

11

Septima Solutio

$$\begin{array}{ccc|ccc} x+y \propto a & y+z \propto b & z+x \propto c & x \propto a-y & y \propto b-z & z \propto b-y \\ y+z \propto b & z+x \propto c & x+y \propto a & x \propto c-z & y \propto a-x & z \propto c-x \\ \hline 2y+x+z \propto a+b & 2z+y+x \propto b+c & 2x+y+z \propto a+c & & & \end{array}$$

His sic ordinatis tribus hinc 3 diversis modis desiderata eliciemus.

$$y \propto \frac{a+b-x-z}{2} \propto \frac{b-z}{1} \propto y$$

$$\cancel{a+b}-x-z \propto \cancel{2b}-2z$$

$$a-x \propto b-z$$

Hinc x

$$x \propto a-b+z \propto c-z \propto x$$

$$z \propto \frac{b+c-a}{2}$$

Hinc z

$$z \propto b-a+x \propto c-x \propto z$$

$$x \propto \frac{a+c-b}{2}$$

2°

$$z \propto \frac{b+c-y-x}{2} \propto \frac{b-y}{1} \propto z$$

$$\cancel{b+c-y}-x \propto \cancel{2b}-2y$$

$$c-x \propto b-y$$

Hinc x

$$x \propto c-b+y \propto a-y \propto x$$

$$y \propto \frac{b+a-c}{2}$$

Hinc z

$$y \propto b-c+x \propto a-x \propto y$$

$$x \propto \frac{a+c-b}{2}$$

3°

$$x \propto \frac{a+c-y-z}{2} \propto \frac{a-y}{1} \propto x$$

$$\cancel{a+c-y}-z \propto \cancel{2a}-2y$$

$$c-z \propto a-y$$

Hinc x

$$y \propto a-c+z \propto b-z \propto y$$

$$z \propto \frac{b+c-a}{2}$$

Hinc z.

$$z \propto c-a+y \propto b-y \propto z$$

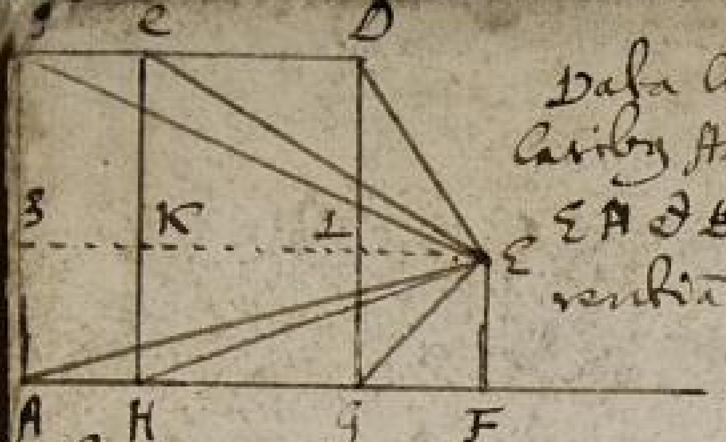
$$y \propto \frac{b+a-c}{2}$$

Hinc sequens Theorema fit perspicuum.

$$a+z \propto b+x \quad c+y \propto b+x \quad a+z \propto c+y:$$

$$\begin{array}{ccc} x & y & z \\ 2 & 3 & 4 \\ 5 & 7 & 6 \\ a & b & c \end{array}$$

12



89

Data linea indefinita AF ductisque in ea tribus perpendicularibus AB, CH et GD, invenire punctum E ita ut ductam lineam EA et BE, EH et EC, EG et ED quadrata aequalem habeant differentiam. Sit BA $\propto a$, AH $\propto b$, HG $\propto c$ ∴ BI $\propto a-y$, GF $\propto x$, FE $\propto y$

35

$b+c+x$
$b+c+x$

$cc+2cx+xx+aa-2ay+yy \propto \square$ CE
$cc+2cx+xx+yy \propto \square$ EH } sub.

$2ay-aa$

$bb+2bc+2bx+2cx+xx+cc+aa-2ay+yy \propto \square$ BE
$bb+2bc+2bx+2cx+xx+cc+yy \propto \square$ AE } sub.

$2ay-aa$

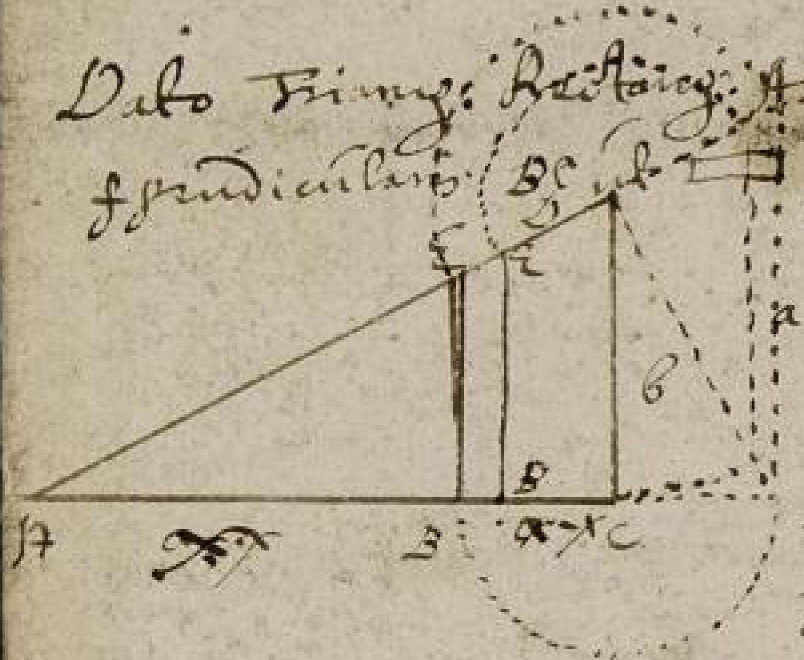
$xx+aa-2ay+yy \propto \square$ DE
$xx+yy \propto \square$ GE } sub.

$2ay-aa$

Hinc patet hoc fieri ad punctum ubicunque pluribus perpendicularibus ductis.

90.

Dato triangulo rectangulo ADC punctum invenire in AC, B, a quo ductis perpendicularibus BE in CD $\propto$ sit vel ▭ ABC / AED: (applicatione)



$a + b + x + \frac{bx}{a} \propto$ BE
$+ b$ multi.

$\frac{ax-xx}{1} \propto \frac{bbx}{a}$

$aax-axx \propto bbx$
$aa-ax \propto bb$

$\frac{aa-bb}{a} \propto x$ et $a-x \propto \frac{bb}{a} \propto$ BC

Sit AD $\propto c$, AE $\propto x$, ED $\propto c-x$

$c + b + x + \frac{bx}{c} \propto$ EB multi. $+ b$ erit $\frac{bbx}{c} \propto \frac{cx-xx}{1}$

∴ $c-x \propto \frac{bb}{c}$

$bbx \propto ccx-xxc$
$bb \propto cc-cx$

$\frac{cc-bb}{c} \propto x$

91

Quaeritur qualis sit linea AEC hanc proprietatem habens ita ut ▭ AED sit $\propto \square$ a dimidia linea EC,

Sit FE $\propto x$
et GC $\propto y$
∴ AE $\propto a+x$
∴ ED $\propto a-x$

$\frac{aa-xx}{1} \propto \frac{yy}{4}$

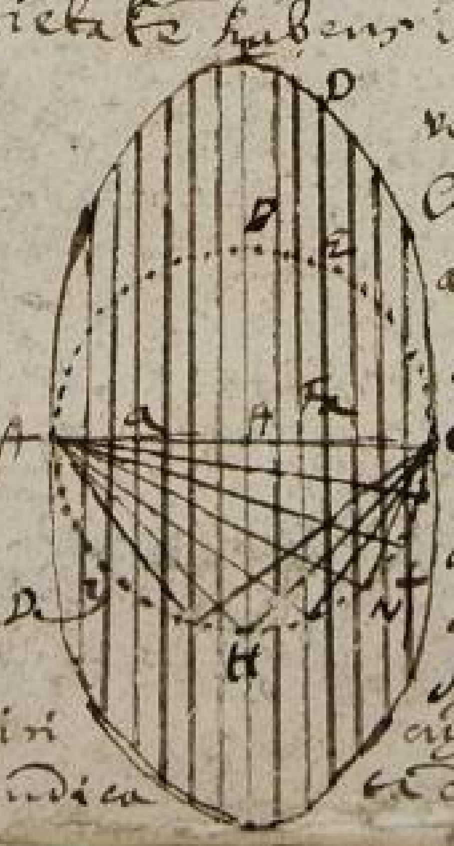
$4aa-4xx \propto yy$

Sit $x \propto 0$ et
erit $4aa \propto yy$
$2a \propto y \propto$ FB

Sit $y \propto 0$ et
erit $4xx \propto 4aa$
$x \propto a$

Hinc patet hanc lineam esse ellipsin cuius constructio ex sequentibus patet

vel describatur circulus ABGH assumaturque AB p. BC, FE p. ED atque G. sic de ceteris, ubi patet si supponatur semper $\propto \square$ a tertia vel quarta parte aliud genus ellipsium cuius constructio nullam difficultatem:

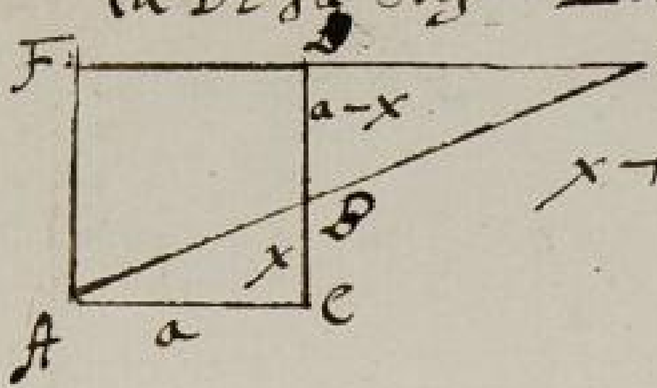


II AEB

hinc orir. a jam indicata

13

Dato □to AFDB ex A in producta DE linea AE ducere ita ut □ tri DE sit dupl. □ti AFDB



E prop: A simil: ABC et EBD erit ut

$x \;\pi\; a \;\pi\; a-x \;\pi\; \frac{aa-ax}{x} \propto DE$

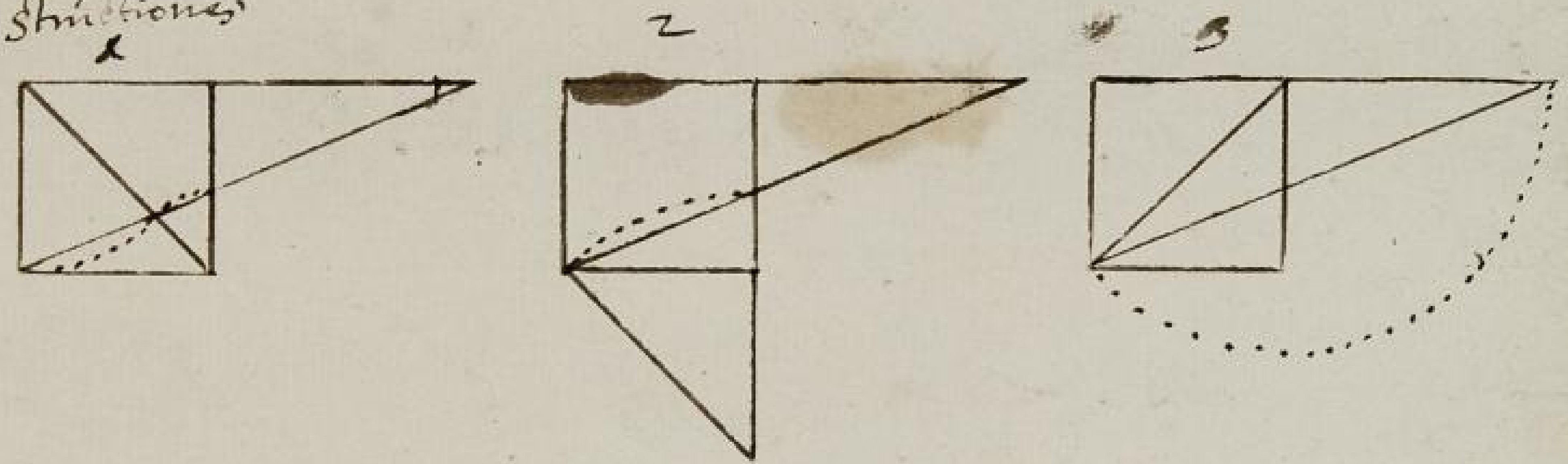
$\frac{2aa}{1} \propto \frac{a^4 - 2a^3x + aaxx}{xx} \propto$ □ DE

$\cancel{2}xx \propto aa - 2ax + \cancel{xx}$

$xx \propto -2ax + aa$

$x \propto -a + \sqrt{2aa}$ et tunc sit DE ∝ x :: ∝ $\sqrt{2aa}$

sit et DB ∝ x :: juxta hanc inventa $x \propto 2a - \sqrt{2aa}$. hinc triplices con-structiones



Dato □to AFDB ex A in producta DE linea AE ducere ita ut latus AC sit media proportionalis inter CB et FE:

aspice praecedens schema: prop: A simi: ABC et EBD erit ut

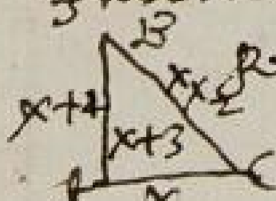
$x \;\pi\; a \;\pi\; a-x \;\pi\; \frac{aa-ax}{x} \propto DE$ add: a

erit $\frac{aa}{x} \propto FE$ ac proinde

$x \;\pi\; a \;\pi\; a \;\pi\; \frac{aa}{x}$ et erit $aa \propto \frac{aax}{x}$ s. $aa \propto aa$. hinc patet

duplex methodus datis duabus lineis tertiam continue proportionalem inveniendi prima quidem cum media est major altera data, secunda cum med. minor.

Invenire Triang: Rectang: ABC cujus latera et area sunt in tali pro-portione ut si AC sit 5 AB sit 6 BC sit 7 Area vero 8:



sit AC ∝ x :: AB ∝ x+1 BC ∝ x+2 Area ∝ x+3.

$\begin{cases} xx+2x+1 \\ xx \end{cases}$

$\cancel{2}xx + \cancel{2x} + 1 \propto \cancel{x}x + \overset{2}{\cancel{4}}x + \overset{3}{\cancel{4}}$

$xx \propto 2x + 3$

$x \propto 1 + \sqrt{3}$

$x \propto 3$

vel secundo

$x+1$ simul
x

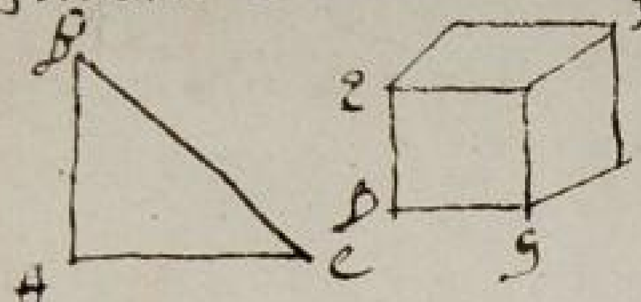
$\frac{xx+x}{2} \propto \frac{x+3}{1}$ $\quad xx + \cancel{x} \propto \cancel{2}x + 6$

$xx \propto x + 6$

$x \propto \frac{1}{2} + \sqrt{\frac{1}{4}+6}$ s. $x \propto 3$:

105

Invenire Cubum EDGF cujus superficies $\propto$ sit superficiei dati Trianguli ABC linea vero hujus Cubi simul juncta $\propto$ sit ambitui Trian=guli ABC



sint latera Triang: rationalia hoc e AC $\propto$ 3a AB $\propto$ 4a BC $\propto$ 5a

DE latus Cubi x unde superficies Cubi 6xx $\propto$ 6aa area Triang: ABC

linea Cubi x $\propto$ a

Ido 12x $\propto$ 12a ambitus ABC

x $\propto$ a hinc patet.

Si trianguli latera sint			Latus Cubi fore
3	4	5	1
6	8	10	2
9	12	15	3
12	16	20	4

atque sic porro sine fine.

106

Quaere numeros ita ut producta trium se multiplicantium sint $\propto$ datis numeris a, b, c, d:

105

Quaerere quatuor numeros, ita ut producta trium se multiplicantium sint aequalia datis numeris a, b, c, d.

Sit primus x, 2dus y, 3us z, 4tus u; ergo $xyz = a$, $yzu = a$, $zux = c$, $uxy = d$

$xyz = a$, $yzu = b$ mult:
$xyyzzu = ab$
M: $zux = c$
$xxyyuuz^3 = abc$
D: $yxy = d$
∴ $xxyyuu = dd - xxyyuu = dd$

vel brevius invenitur z, dividendo $z^3 = \frac{abc}{dd}$

tunc $z = \sqrt[c]{\frac{abc}{dd}}$, eadem ratione invenies $y = \sqrt[c]{\frac{abd}{cc}}$, $x = \sqrt[c]{\frac{acd}{bb}}$, $u = \sqrt[c]{\frac{bcd}{aa}}$

e xyz sit z proveniunt xy, item yzu sit z provenit yu, hoc multiplicatum in xy provenit $xuyy$ quod divisum per xuy exhibet y; invento jam y et z, dividendo e xyz per yz habebis x, eodem dividendo zux per zx obtinebis u.

Sit $a = 60$, $b = 120$, $c = 90$, $d = 72$ erunt numeri desiderati 3 4 5 6.

106.

Tres numeros invenire ita ut summa duorum se sequentium multiplicata per reliquum producat numeros a, b, c datos.

ponantur pri. $= x$, 2dus $= y$, tertius $= z$; tunc $xz + yz = a$, $yx + zx = b$, $yx + yz = c$; add.:

$2xz + 2yz + 2yx = a + b + c$
$xz + yz + yx = \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$
S: $xz + yz = a$
$yx = -\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c = d$ #

eadem ratione invenies $yz = \frac{1}{2}a - \frac{1}{2}b + \frac{1}{2}c = e$, $xz = \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c = f$

jam $yx = d$, $yz = e$ M
$yyxz = de$
$xz = f$
$yy = \frac{de}{f}$
tunc $y = \sqrt{\frac{de}{f}}$ eadem ratione invenies

$y = \sqrt{\frac{df}{e}}$ et $z = \sqrt{\frac{ef}{d}}$ sit $a = 20$, $b = 14$ et $c = 18$ erunt desiderati 2. 3. 4.

107

Dati sunt primi et secundi ex tribus numeris summa a, et secundi et tertii b, tertii et primi c.

$x + y = a$
$y + z = b$ Add:
$z + x = c$
$2x + 2y + 2z = a + b + c$
$x + y + z = \frac{a+b+c}{2}$ sub
$x + y = a$
$z = \frac{b+c}{2} - a$ eadem ratione caeteri invenientur.

Aliter

$x + y = a$, $y + z = b$, $z + x = c$
$y = a - x = b - z = y$
17.
$z = b + x - a = c - x$
$x = \frac{a+c-b}{2}$

108

Quatuor Personae debent certā pecuniae summā, 1us, 2us et 3us 180, 2us: 3us: 4us: 240 3us: 4us: pri: 210, 4us, pri: et secun: 150 quaeritur quantum unusquisque debuit? Sint x, y, z et w

$x+y+z = 180$

$y = 180 - x - z$

add. $z + w$

$180 - x + w = 240$

$w = 60 + x$ #

\# $w = 60 + x$

$z + x$

$60 + 2x + z = 210$

$z = 150 - 2x$

$w = 60 + x$

$y = 180 - x - z$

x

$240 + x - z = 150$

$-z \cancel{x}\, 2x - 150$

$90 + 3x = 150$

$3x = 60$

$x = 20$

$y = 50$

$z = 110$

$w = 80$

A
$$\left\{\begin{array}{l} x+y+z = 180 \\ y+z+w = 240 \\ z+w+x = 210 \\ w+x+y = 150 \end{array}\right.$$

$3x+3y+3z+3w = 780$

$x+y+z+w = 260$

Aliter

$x+y+z+w = 260$

Sub: $y+z+w = 180$

$x = 20$ et eadē ratione alios.

Aliter

$x+y+z = 180$, $y+z+w = 240$ $z+w+x = 210$ $w+x+y = 150$

$\cancel{x+y = 180 - z = 150 - w = x+y}$

$\cancel{z = w - 30 = 240 - w = y+z}$

$\cancel{2w + y = 270}$

$x+y = 280 - z = 150 - w = x+y$ | $y+z = 180 - x = 240 - w = y+z$

$z = w + 30$

$w = z - 30$

$w = 60 + x$

$x = w - 60$

$x+z = 180 - y = 210 - w = x+z$, $\cancel{z}+w = \cancel{240} - y = 210 - x = z+w$

$y = w - 30$

$w = y + 30$

$x = y - 30$

$y = x + 30$

$x+w = 150 - y = 210 - z = x+w$ $y+w = 150 - x = 240 - z = y+w$

$z = y + 60$

$y = z - 60$

$z = x + 90$

$x = z - 90$

18

SLUB
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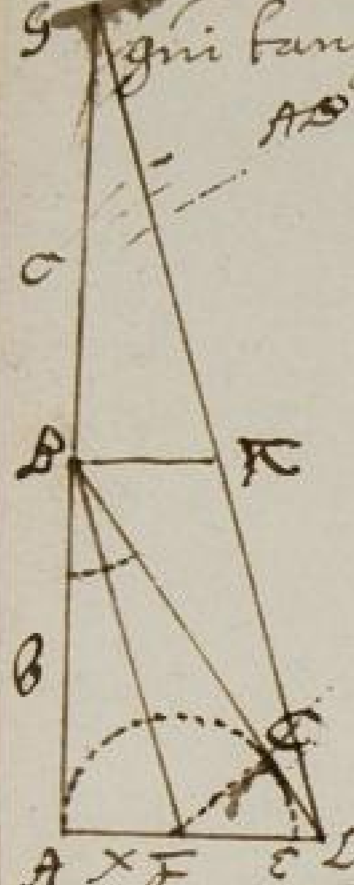
~~Dato triang:~~ Rectang: describere intra illud semicirculu ACE qui tangat latus BD in C:



$AB = b$ $AD = a$ $BD = c$ Prop: 4 simil: ABD & CFD dio.

$AF. s: FC: FE = x$ $b : c :: x : a-x$

$\therefore FD = a-x$

$cx = ab - bx$

$cx + bx = ab$

$x = \frac{ab}{c+b}$

$\overset{GA}{c+b} : \overset{AD}{a} :: \overset{BA}{b} : \overset{AF}{x}$

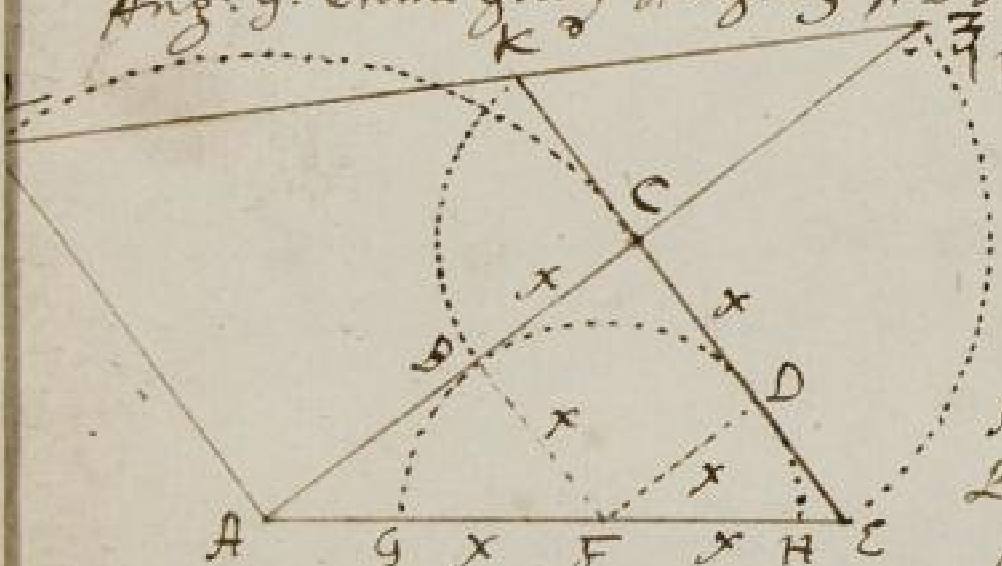
hinc $a - x = \frac{ac}{c+b} = BF$ s: FD Nam $\overset{GA}{c+b} : \overset{AD}{a} :: \overset{GB}{c} : FD$:

hinc 2do $ED = a - 2x = \frac{ac-ab}{c+b}$ hoc est $c+b : a :: c-b : x = ED$:

Hoc est Summa laterum $AB + BD$ ~~est~~ tertiu latus AD sic differentia laterum $BD - BA$ ad ED:

Hinc etiam constat propositi facilis admodu constructio diviso enim angulo ABD bifariam habebimus quaesitu; ~~Cum tunc Ang: ABD sit = ang: duobus oppositis internis G & D, hi autem inter se sub laterum BG & BD, aequalitate scilicet~~ cumulus Ang: ABF = ang: G; & Angul: FBD = ang: BDK hoc est Ang: G: erunt quoq Anguli ABF & FBD inter se aequales:

113



Dato triang: Rectang: ACE, intra ipsu semicirculu GBDH describere tangente latera AC & CE:

$AC = a$ $CE = b$ $AE = c$ $GF. BF. BC. CD. FD. FH = x$

$AB = a-x$ $DE = b-x$

Prop: 4 simil: ACE & ABF erit

$a : b :: a-x : x$

$ax = ab - bx$

$ax + bx = ab$ & $x = \frac{ab}{a+b}$ unde

$\overset{AB}{a+b} : \overset{AL}{a} :: \overset{CB}{b} : \overset{CK}{x}$. #

Hinc $AB = a - x =$ ipsa jam inventa $\frac{aa}{a+b}$ & $DE = b - x = \frac{bb}{a+b}$.

Cum autem prop: 4 sim: sit

$a : c :: x : \frac{cx}{a} = FE$, erit ipsa jam inventa $\frac{bc}{a+b}$ & cum ob eandem causa disposit sint $b : c :: x : \frac{cx}{b} = AF$ erit hoc ibidem $= \frac{ac}{a+b}$

Hinc tertio etiam 1mo $\frac{cx}{a} = FE$ erit $HE = \frac{cx}{a} - x$ hoc est $\frac{bc-ab}{a+b}$ & cum $\frac{cx}{b} = AF$ erit $AG = \frac{cx}{b} - x$ aliqui ideo $= \frac{ac-ba}{a+b}$

Hinc etiam diversa Theoremata 1 $ax + bx = ab$: 2 $aa = \overset{x=AB}{ax+bx}$ 3 $bb = \overset{DE=x}{ax+bx}$

4 $bc = \overset{FE=x}{ax+bx}$, 5 $ac = \overset{x=AF}{ax+bx}$ 6 $bc = \overset{x=HE}{ax+bx+ab}$ 7 $ac = \overset{x=AG}{ax+bx+ab}$:

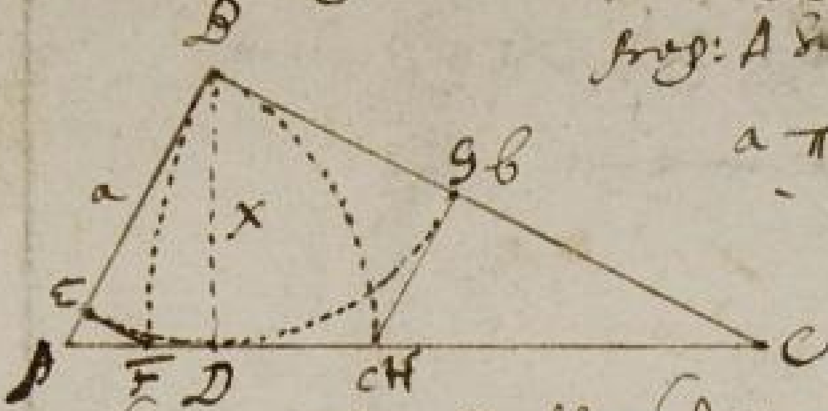
Tandem etiam hinc constat quaratione Dato Rectang: 4 inscribi possit Ultu communem ~~angul~~ Rectang: 4 angulu ACE habens:

23

Dato Triang: Δ invenire perpendiculum BD.

Prop: Δ sim: ABD & BDC hoc est

a π c π x π b



cx ∞ ab Theorema:

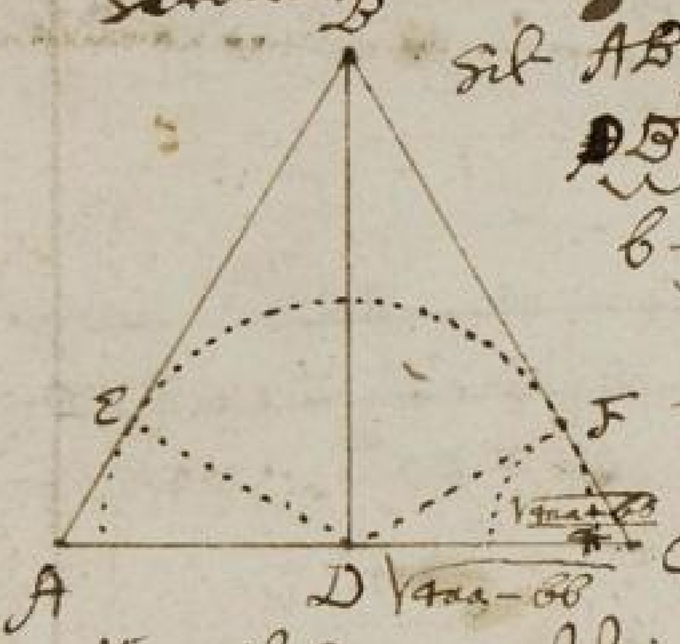
Dx ∞ $\frac{ab}{c}$ hoc est ut latus maximum recto angulo oppositum ad alterum ex reliquis sic tertium ad quaesitum. Sunt ut

$\overset{AC}{c}$ π $\overset{BC}{b}$ π $\overset{a}{AH}$ π $\overset{BG}{x}$ Si: $\overset{AC}{c}$ π $\overset{AB}{a}$ π $\overset{CF}{b}$ π $\overset{BD}{x}$

115

Dato Triang: aequilatero super medio unius lateris describere Semicirculum qui reliqua latera tangat.

Sit AB ∞ 2a BD ∞ b DF ∞ x hinc propter Δ similia DBC & FDC erit ut



b π 2a π x π a FC autem cum sit ∞ √aa−xx erit juxta inventa $\sqrt{aa-\frac{bb}{4}}$ ∞ FC

ab ∞ 2ax

$\frac{b}{2}$ ∞ x

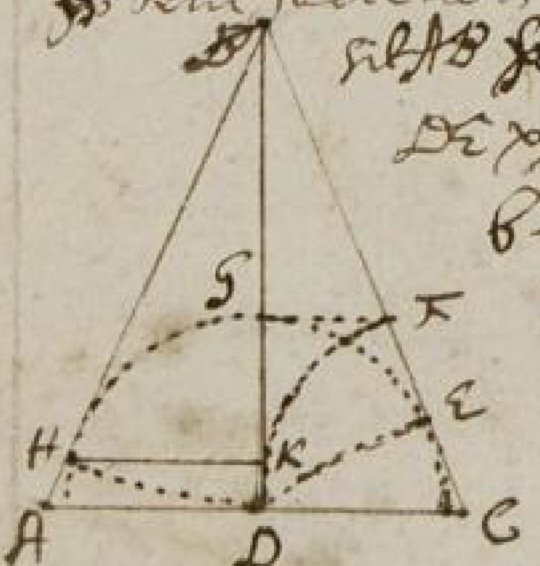
cujus ibidem constructione in praesenti schemate et facilis et manifesta

Hinc BF ∞ erit 2a − $\sqrt{aa-\frac{bb}{4}}$ si: FC ter simplo.

Hinc ibidem patet in triang: Rectang: BDC demissa perpendiculari DF □ DF in BC ∞ □lo BDC: cujus ibidem demonstratio ex aspectu patet; si n. DF multiplicet BC duplum proveniet Δ BDC ac proinde Area Δ ABC. sic producet BD in DC multiplicata; ac propterea □la haec juxta x axi: ∞ erunt. Q.E.D.

116

Idem faciendi sit propositum Triangulum Aequilaterum:



Sit AB vel BC ∞ a DC ∞ c BD ∞ b

DE ∞ x prop Δ simil: DBC & EDC erit ut

b π a π x π c

ax ∞ bc $\overset{DC}{a}$ π $\overset{BD}{b}$ π $\overset{CF}{c}$ π $\overset{DG}{x}$

x ∞ $\frac{bc}{a}$

vel $\overset{BA}{a}$ π $\overset{AD}{c}$ π $\overset{DH}{b}$ π $\overset{HK}{x}$: Differentiae lat.

Hinc prop constat BG π GD hoc est ut BC ad DC ad ∞ BG

DC. hoc est a−c π c; Nam b − $\frac{bc}{a}$ π $\frac{bc}{a}$ π a−c π c

$\frac{abc-bcc}{aa}$ ∞ $\frac{abc-bcc}{a}$

Sit nunc BF π FC π BG π GD

Hoc est a−c π c π · π

x 3 5 7 9 11 13 15 &

x 4 9 16 25 36 49 64.

erunt quadrati

Patet itaque quodlibet quadratum ex tot imparibus numeris componi, quot unitates continet eius latus

Si vero progressio fuerit x 4 7 10 13 16 19 &c.

erunt pentagonales x 5 12 22 35 51 70.

atque sic porro.

Hinc dato angulorum numero si ex eo auferatur binarius relinquetur progressionis excessus ex qua videlicet progressione datus componitur multangulus.

Modus autem quo ex dato latere inveniatur multangulus talis est.

Intelligatur datum latus esse x eiusque triangulus ab ipso conflatus et dato illud latus $= \frac{xx+1x}{2}$ et eiusdem quadratus $\frac{2xx+0x}{2}$ hoc est xx

pentagonus $= \frac{3xx-x}{2}$ et hexagonus $= \frac{4xx-2x}{2}$ heptagonus $= \frac{5xx-3x}{2}$

atque sic porro in infinitum

Unde si latus sit 12 et quaeratur eiusdem triangulus erit ipse 78 et pentagonus 210 et hexagonus 276. Quod si latus datum 38 erit eiusdem pentagonus 2147 sin vero 39 pentagonus 2262.

Unde liquet dato latere invenire polygonum

Etenim si quadratum lateris ducatur in progressionis excessum et a producto auferatur id quod fit a latere dato in progressionis excessum binario multatum, residuum erit duplum polygoni, hoc quoque idem est quod supra traditur atque hoc modo etiam in formam canonis referri potest:

Canon

Ducito datum latus in progressionis excessum, a producto aufer eundem excessum multatum binario et residuum duc in datum latus, fietque duplum polygoni

Dato autem polygono invenire eius latus fiet contrario

numerum $\frac{1xx+1x}{2} = 78$ hoc est $xx = -1x + 156$

quare $x = -\frac{1}{2} + \sqrt{\frac{1}{4}+156}$ hoc est $x = 12$

similiter ad inveniendum latus seu radicem pentagonalem ex 2147:

erit $\frac{3xx-1x}{2} = 2147$ sive $xx = \frac{1x+4294}{3}$ unde $x = \frac{1}{6} + \sqrt{\frac{1}{36}+\frac{4294}{3}}$ hoc est $x = 38$.

Hinc jam patet dato numero militum lateris aciei secundum aliquam figuram regularem ordinatae invenire numerum totius aciei.

Ratio inveniendi summam numerorum quadratorum ab unitate:

Sit exempli gratia: invenienda summa 12 primorum quadratorum quorum primum est 1 et ultimum 144:

Ad 12 quadratorum numerum addita unitate multiplicetur summa 13 per numerum multitudinis 12 et fit 156 deinde additis 12 et 13 duc summam 25 in productum inventum, et quod exsurgit divide (semper) per 6 oritur 650 quaesita summa

~~Quod si vero quaeratur summa partium quadratorum ab~~

Ratio inveniendi: tali modo cuborum summam:

Sint 12 cubi primi ab unitate, quadretur primorum summa eorundem laterum quae est 78 haec enim quadrata dat 6084 summam quaesitam.

257:

Invenire duos numeros in ratione 2 ad 5 eandemque multitudinem partium habentes, ita ut aggregatum partium aliquotarum majoris numeri sit 24 major quam aggregatum partium minoris.

Esto $a = 2$ Major numerus quaesitus $= bxx$ Jam constat axx ad bxx
$b = 5$ minor quaesitus $= axx$ esse ut 2 ad 5 et praeterea
$c = 24$
multitudinem partium aliquotarum eandem esse etenim ipsius axx partes aliquotae sunt $1.\ a,\ x.\ ax.\ axx$
at vero ipsius bxx sunt $1.\ b.\ x.\ bx.\ xx$ restat ut aggregatum partium aliquotarum majoris bxx sit $24 = c$ major quam aggregatum partium aliquotarum ipsius axx hinc jam exsurgit talis aequatio

$$\cancel{1} + b + bx \cancel{+x} \cancel{+xx} = \cancel{1} + a + c \cancel{+x} + ax \cancel{+xx}$$

$$bx - ax = a + c - b$$

$$x = \frac{a+c-b}{b-a}$$ Hinc $\begin{array}{r} xx = 49 \\ 2 \\ \hline 98 \end{array}$ = minor $\begin{array}{r} xx = 49 \\ 5 \\ \hline 245 \end{array}$ = major.

258.

Invenire duos numeros, quorum major sit 3 plus quam minor ita ut si ad eorundem productum addatur major numerus, summa sit quadratum

Ponatur major $x+3$ et minor x

$$\begin{array}{l} \quad x+3 \\ \quad\ \ x \\ \hline \text{Productum } xx+3x \\ \text{Addi: major } x+3 \\ \hline xx+4x+3 = 9-6x+xx \\ 4x = 6-6x \\ \qquad\ \ +6x \\ \hline 10x = 6 \\ x = \frac{3}{5} \end{array}$$

cum autem haec summa ○ sit □tum adaequetur □to. illud autem ex latere $3-x$, ubi pro quolibet numero ad libitum assumpto $-x$ cujus □tum est #

$x = \frac{3}{5}$ minor unde major erit $3\frac{3}{5}$

[illegible]

259

Dividere 10 in duas partes alias quam 3 et 7 quarum major multiplicata per minoris quadratum faciat 63

Sumatur minor pars $x+3$ ergo major $7-x$

$$\begin{array}{l} x+3 \\ x+3 \\ \hline \square \text{ minoris } xx+6x+9 \\ \text{Multiplic: per majorem } -x+7 \\ \hline -x^3+xx+33x+\cancel{63} \infty \cancel{63} \\ \hline \end{array}$$

$$xx+33x \infty x^3$$

$$x+33 \infty xx$$

$$x \infty \tfrac{1}{2}+\sqrt{\tfrac{1}{4}+33}$$

Subt. 10 / $\sqrt{33\tfrac{1}{4}}+3\tfrac{1}{2}$

Unde erit $x+3 \infty 3\tfrac{1}{2}+\sqrt{33\tfrac{1}{4}}$ et reliq: $\infty\ 6\tfrac{1}{2}-\sqrt{33\tfrac{1}{4}}$ pro majore parte

260

Duobus numeris datis invenire alium qui utrique seorsim additus faciat quadratum. Sunto dati numeri a et b numerus addendus $\infty\ xx-a$

Jam manifestum est si ad a ~~[illegible]~~ addatur $xx-a$ relinqui xx quadratum restat tantum ut $xx-a+b$ faciat □tum verum cum hoc o fit adaequetur quadrato quod fit a latere $c-x$

$$\text{Itaque } \cancel{xx}-a+b \infty cc-2cx+\cancel{xx}$$

$$\text{Ergo } 2cx \infty cc+a-b \text{ et } x \infty \frac{cc+a-b}{2c}$$

261

Invenire duos numeros, quorum □ta una cum rectangulo ab ipsis ambobus faciant □tum. Esto unus numerus $\infty\ a$ alter $\infty\ x$

requiritur itaque ut $aa+xx+ax$ sit □tum quod cum ita o fit adaequetur quadrato a latere quocunque sit: $-b+x$

$$\text{Eritque } aa+\cancel{xx}+ax \infty \cancel{xx}-2bx+bb$$

$$aa+ax \infty -2bx+bb$$

$$2bx+ax \infty bb-aa$$

$$x \infty \frac{bb-aa}{2b+a}$$

262

Datis duobus numeris cubis invenire alios cubos quorum summa ∞ sit differentia datorum: Sint: cubi a^3 et b^3 a^3 major b^3 minor

Latus quaerendi cubi $\infty\ a-x$ et alterius $\infty\ \frac{aax}{bb}-b$ huius □tum

$\infty\ \frac{aaxx}{b^4}-\frac{2aax}{b}+bb$ et eritque Cubus secundi lateris ∞

Cubus multiplic. per $\frac{aax}{bb}-b$ $\left.\begin{array}{l} \frac{a^6x^3}{b^6}-\frac{3a^4xx}{b^3}+3aax-b^3 \\ \text{Cubus lateris pri: } a^3-3aax+3axx-x^3 \end{array}\right\}$ adda:

Diff: datorum $= a^3 - b^3 = \frac{a^6x^3}{b^6} - \frac{3a^4xx}{b^3} - b^3 + a^3 + 3aaxx - x^3 =$ summ. cub: quaesi-

hoc est $0 = a^6x^3 - 3a^4b^3xx + 3ab^6xx - b^6x^3$ et delet. ubique xx

ac transpositis terminis erit

$a^6x - b^6x = 3a^4b^3 - 3ab^6$ et $x = \frac{3a^4b^3 - 3ab^6}{a^6 - b^6}$ et abbrevia-

ta fractio per $a^3 - b^3$ remanet $x = \frac{3ab^3}{a^3 + b^3}$ unde est $a - x = \frac{a^4 - 2ab^3}{a^3 + b^3}$

latus primi cubi et $\frac{aax}{bb} - b = \frac{2a^3b - b^4}{a^3 + b^3}$ latus secundi cubi:

Corollarium et hic licet invenire alios cubos numeros quorum 3 simul additi faciant 4tum seu quorum major sit 3bus reliquis aequalis.

263

Datis duobus cubis, invenire duos alios cubos quorum differentia sit aequalis summae datorum, sint dati cubi a^3 et b^3

Sit latus primi cubi $a + x$ secundi $\frac{aax}{bb} - b$

Cubus lateris primi $a^3 + 3aax + 3axx + x^3$

Sec: cub: lateris sec: $\frac{a^6x^3}{b^3} - \frac{3a^4xx}{b^3} + 3aax - b^3$

Different: cub: $= a^3 + 3axx + x^3 - \frac{a^6x^3}{b^6} + \frac{3a^4xx}{b^3} + b^3 = a^3 + b^3 =$ summ. datorum.

et transpositis et deletis ubique xx erit $\frac{a^6x}{b^6} - x = \frac{3a^4}{b^3} + 3a$ et $a^6x - b^6x = 3a^4b^3 + 3ab^6$

Hoc est $x = \frac{3a^4b^3 + 3ab^6}{a^6 - b^6}$ seu abbreviata fractione per $a^3 + b^3$

erit $x = \frac{3ab^3}{a^3 - b^3}$ unde est $a + x = \frac{a^4 - 2ab^3}{a^3 - b^3}$ et $\frac{aax}{bb} - b = \frac{b^4 + 2ab^3}{a^3 - b^3}$

Corollarium

Sic neque licet invenire alios cubos, quorum major sit tribus reliquis aequalis

264

Datis duobus cubis invenire duos alios cubos quorum differentia sit aequalis differentiae datorum. Sint dati cubi a^3 major b^3 minor latera quaesitorum sint $x - b$ et $\frac{bbx}{aa} - a$

Cubus lateris minoris $\frac{bbx}{aa} - a = \frac{b^6x^3}{a^6} - \frac{3b^4xx}{a^3} + 3bbx - a^3$ } minus subtrahatur a majori

Cubus lateris majoris $x - b = x^3 - 3bxx - \cancel{3bbx} - b^3$

eritq; differentia cuborum $x^3 - 3bxx - b^3 - \frac{b^6x^3}{a^6} + a^3 = a^3 - b^3$ diff: datorum

et $x^3 - \frac{b^6x^3}{a^6} = 3bxx - \frac{3b^4xx}{a^3}$ utriq; deletis xx et translatis sub eadem

denominatione est $a^6x - b^6x = 3a^6b - 3a^3b^4$ unde $x = \frac{3a^6b - 3a^3b}{a^6 - b^6}$

et abbreviando per $a^3 - b^3$ erit $x = \frac{3a^3b}{a^3 + b^3}$ hinc $x - b = \frac{2a^3b - b^4}{a^3 + b^3}$ et $\frac{bbx}{aa} - a$

$= \frac{2ab^3 - a^4}{a^3 + b^3}$ latus minoris. Corollarium

Sic licet invenire 2 cubos ut bini cubi sint binis cubis aequales

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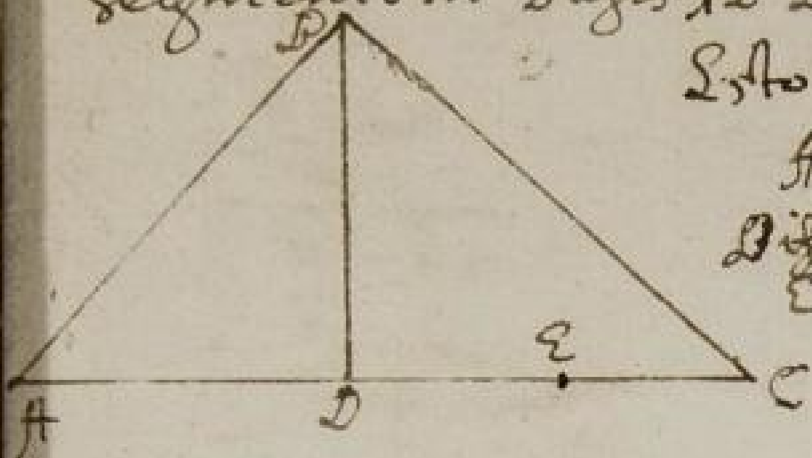
Dato uno latere circa rectum B trianguli ABC et differentia segmentorum basis AD DC invenire latera



Esto AC ∞ x $\therefore$ AE ∞ x − b Nota omnium
AB ∞ a   AD ∞ $\frac{1}{2}x - \frac{1}{2}b$ horum determinata
Diff: ∞ b   DC ∞ $\frac{1}{2}x + \frac{1}{2}b$ quantitatibus.
EC

jam propter Triang: sim: ABD et ACB
ut AB ad AD sic AC ad AB
est a $::$ $\frac{x-b}{2}$ $::$ x $::$ a

$\therefore$ $aa \infty \frac{xx - bx}{2}$

$2aa \infty xx - bx$

unde quoque $xx \infty bx + 2aa$ et consequens
$x \infty \frac{1}{2}b + \sqrt{\frac{1}{4}bb + 2aa}$

vel aliter:

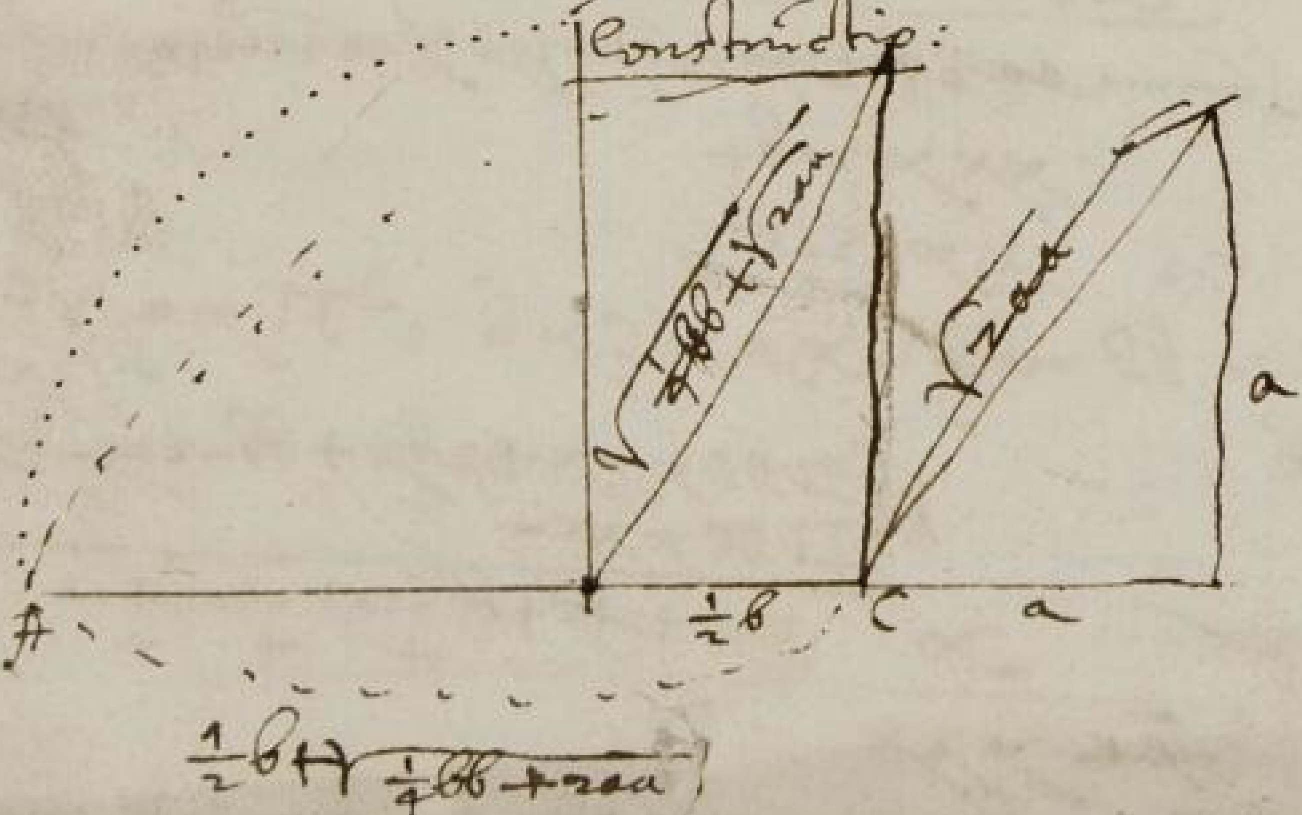
cum □tum AB minus □to AD fit ∞ □ BD hoc est □ BD est ∞ $aa - \frac{1}{4}xx + \frac{1}{2}bx - \frac{1}{4}bb$ et huic aequale fit ▭ ADC ∞ $\frac{1}{4}xx - \frac{1}{4}bb$ erit aequatio inter haec duo scilicet

$aa - \frac{1}{4}xx + \frac{1}{2}bx - \cancel{\frac{1}{4}bb} \infty \frac{1}{4}xx - \cancel{\frac{1}{4}bb}$

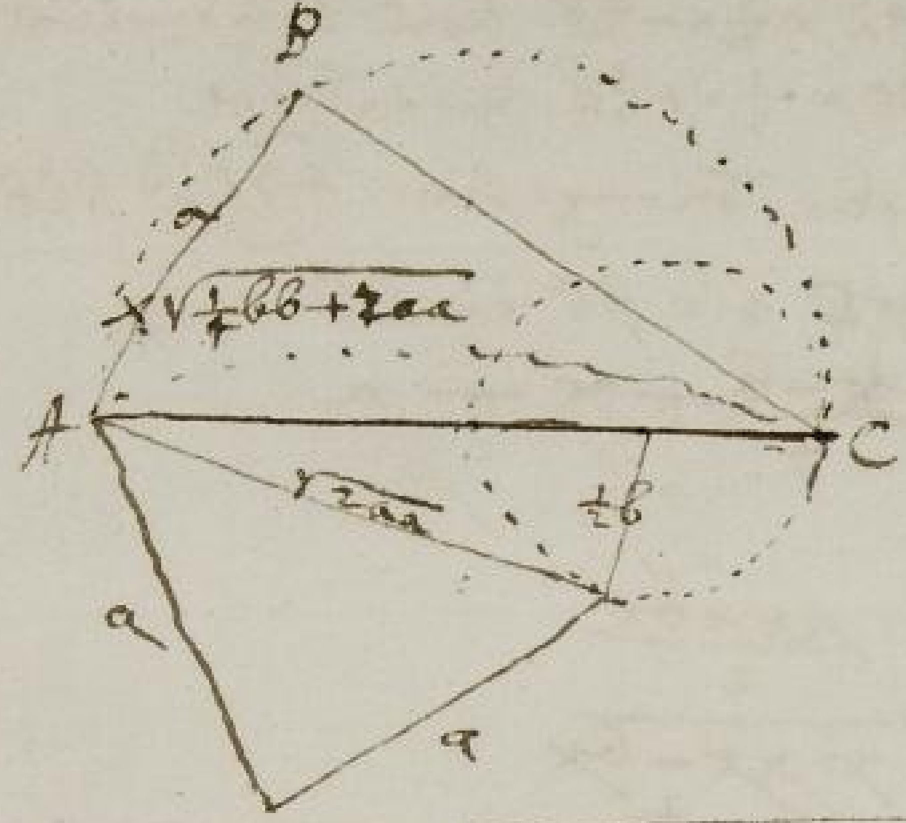
adeoque $aa + \frac{1}{2}bx \infty \frac{1}{2}xx$ hoc est reducta aequatione

$2aa + bx \infty xx$ seu $x \infty \frac{1}{2}b + \sqrt{\frac{1}{4}bb + 2aa}$

Constructio:

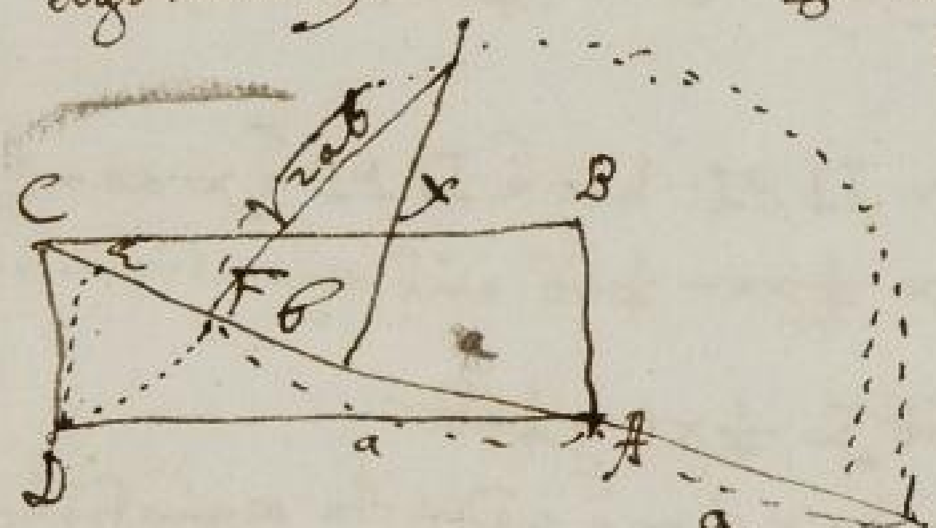


41



$2a-b$
b
$2b-bb$
$2ab-bb$
$+bb$
$2ab$

Parallelogrammo rectangulo datis duobus excessibus AF et EC quibus diagonalis excedit utrumque latus invenire latera singula



Sit $AF \propto a$
$EC \propto b$
$EF \propto x$
$\therefore AE$ f: $AD \propto a+x$
$\therefore CF$ f: $CD \propto b+x$
per 47. lib: 1

$a+b+x$
$a+b+x$ } multi:

AD $\begin{cases} \square AD \propto aa+2ax+xx \\ \square DC \propto bb+2bx+xx \end{cases}$

Summa $\cancel{aa}+\cancel{bb}+\cancel{2ax}+\cancel{2bx}+\cancel{2}xx \propto \cancel{aa}+2ab+\cancel{2ax}+\cancel{2bx}+\cancel{bb}+\cancel{xx}$

$\therefore xx \propto 2ab$
$x \propto \sqrt{2ab}$

vel aliter ponendo AB vel AE $\propto x$ EC $\propto b$ et FC $\propto a$ $\therefore$ AF f $x+b-a$
et AC $\propto x+b$

AD $\begin{cases} \square AD \propto xx+2bx+bb-2ax-2ab+aa \\ \square DC \propto xx- \end{cases}$

$\square AC \propto \cancel{bb}+\cancel{2bx}+\cancel{xx} \propto \cancel{2}xx+\cancel{2bx}+\cancel{bb}-2ax-2ab+aa$
$+ \quad +\quad -$

$\therefore 2ax+2ab-\cancel{aa} \propto xx$

$a+\sqrt{\cancel{aa}-\cancel{aa}+2ab}$ et $a+\sqrt{2ab} \propto x$ NB. Haec aequatio est primi casus cum $2ab$ maior sit quam aa

$\therefore$

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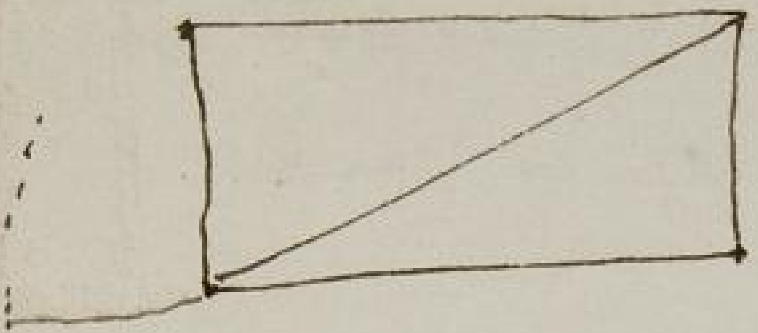
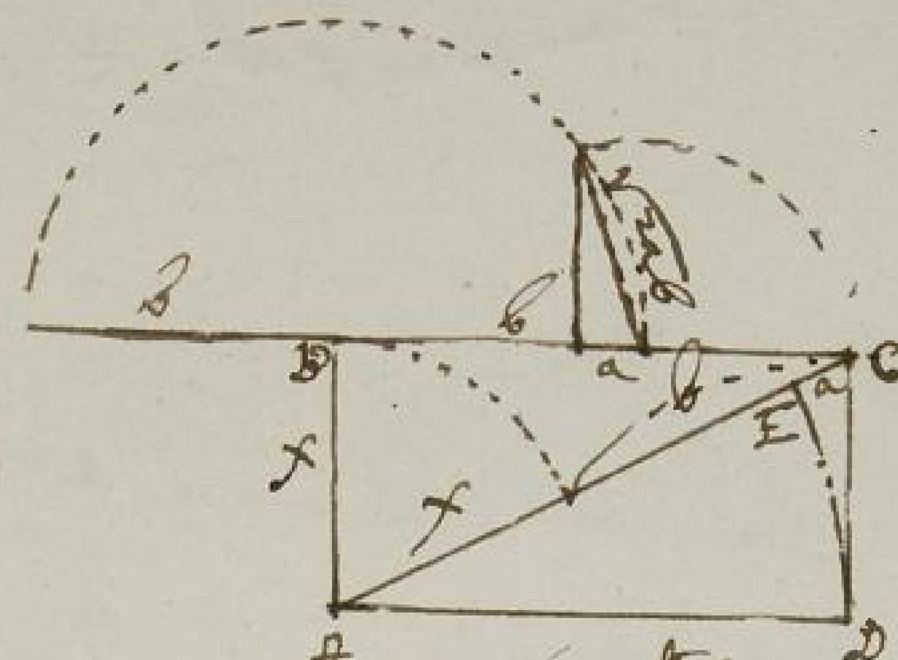




Invenire duos numeros in ratione 2 ad 5 ita ut quad. ratum majoris per minorem multiplicatum faciat 1000

Esto primus $2x$ alter $5x$

jam juxta data □tum majoris $\propto 25xx$ } Mult: per minorem: $2x$

$1000 \propto 50x^3$

$20 \propto x^3$

$\sqrt{c}\,20 \propto x$

Unde cum $x \propto \sqrt{c}\,20$

Multiplic: per 5 hoc est

$\sqrt{c}\,125$

$\sqrt{c}\,20$

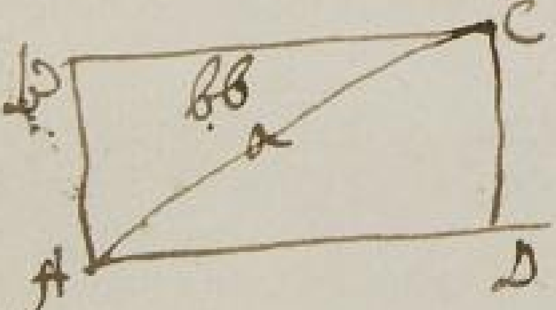
$\sqrt{c}\,2500$ pro majori

et $\sqrt{c}\,20$

$\sqrt{c}\,8$

$\sqrt{c}\,160 \propto$ minori

Dato Diagonali paralle: rectanguli et ejus area invenire singula latera (Quod idem est ac si diceremus invenire duos numeros, qui in se invicem ducti faciant 864 et quorum summa quadratorum $\propto$ 1872:



Esto $AC \propto a$

Area $ABCD \propto bb$

$AD \propto x+y$

AB sive: $DC \propto x-y$ #

jam porro juxta datum rectanguli $ABCD \propto xx-yy \propto bb$ et insuper

Ad { □tum $AD \propto xx+2xy+yy$
□tum $DC \propto xx-2xy+yy$

Summ: □torum $\propto 2xx+2yy \propto aa$

et $xx+yy \propto \frac{aa}{2}$
$xx-yy \propto bb$ } aequalibus aequalia subtrahe

$2yy \propto \frac{aa}{2} - bb$

et $yy \propto \frac{1}{4}aa - \frac{1}{2}bb$

$y \propto \sqrt{\frac{1}{4}aa - \frac{1}{2}bb}$ vel etiam...

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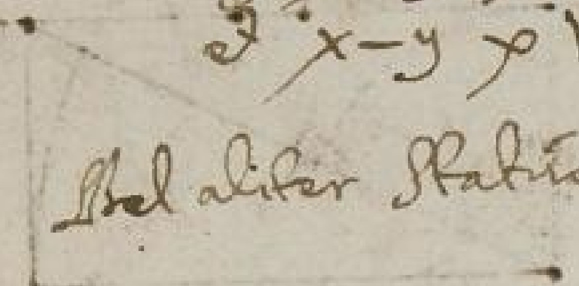
ad $\propto$ inveniendum cum $xx+yy \propto \frac{1}{2}aa$ adda·
aequalibus aequalia scil. $xx-yy \propto bb$

provenit $2xx \propto \frac{1}{2}aa+bb$
et $xx \propto \frac{1}{4}aa+\frac{1}{2}bb$
$x \propto \sqrt{\frac{1}{4}aa+\frac{1}{2}bb}$

$\sqrt{\frac{1}{4}aa+\cancel{\frac{1}{2}bb}}$
$\sqrt{\frac{1}{4}aa-\cancel{\frac{1}{2}bb}}$

$\frac{aa}{4} \times \frac{aa}{4}$ $\frac{2aa}{4}$ $\frac{aa}{2}$

Ergo $x+y \propto \sqrt{\frac{1}{2}aa+\sqrt{\frac{1}{4}a^4-b^4}}$
et $x-y \propto \sqrt{\frac{1}{2}aa-\sqrt{\frac{1}{4}a^4-b^4}}$



Vel aliter statuendo AD $\propto x$
AC $\propto a$
DC $\propto y$

jam multiplicando AD $\propto x$
per DC $\propto y$
productum $\propto xy \propto bb$
$\therefore y \propto \frac{bb}{x}$

Etiam duo latera AD et DC scil: $xx+yy \propto aa$ provenit
restituto valore ipsius yy erit talis aequatio:
$xx+\frac{b^4}{xx} \propto aa$
et $x^4+b^4 \propto aaxx$ unde
$x^4 \propto aaxx-b^4$ et $xx \propto \frac{1}{2}aa \sqrt{\frac{1}{4}a^4-b^4}$
et $x \propto \sqrt{\frac{1}{2}aa+\sqrt{\frac{1}{4}a^4-b^4}}$.

Vel etiam abbreviando ut $b^4 \propto aacc$ fiat ut
$a \;::\; b \;::\; b$ ad $\frac{bb}{a} \propto c$ unde
jam $bb \propto ac$
et $b^4 \propto aacc$
jam assumendo a pro unitate erit loco hujus aequationis scil.
$\cancel{x^4 \propto aaxx-aacc}$
talis $x^4 \propto axx-cc$
et $xx \propto \frac{1}{2}a+\sqrt{\frac{1}{4}aa-cc}$ vel $xx \propto \frac{1}{2}a-\sqrt{\frac{1}{4}aa-cc}$
et $x \propto \sqrt{\frac{1}{2}a+\sqrt{\frac{1}{4}aa-cc}}$ et $x \propto \sqrt{\frac{1}{2}a-\sqrt{\frac{1}{4}aa-cc}}$.

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