Ad pag. 3. linea 12 et sequent.

$\ldots = \sqrt{1-xx}$

Manuscripte des Johannes Origanus

in Leiden aus den Jahren 1644. 1655. u. 1656, Einlagen zu den Briefen des Origanus an seinen ehemaligen Zögling Hans Heinrich von Lest (Schwiegervater von E. W. von Tschirnhaus), damals Kammerjunker des Herzogs Moritz von Sachsen. Die Briefe selbst finden sich in der „Autographensammlung" der Bibl. d. Oberl. Ges. d. W.

Reinhardt.

34 Seiten

beginnender Papierzerfall

Ad pag. 3. linea 12 et sequent.

Quia $z \propto \sqrt{40044 - xx} - \sqrt{1 - xx}$.

De Linea Conchoidis

Geometrische Reihen.

27.

Ad pag. 3. linea 12 et sequentes

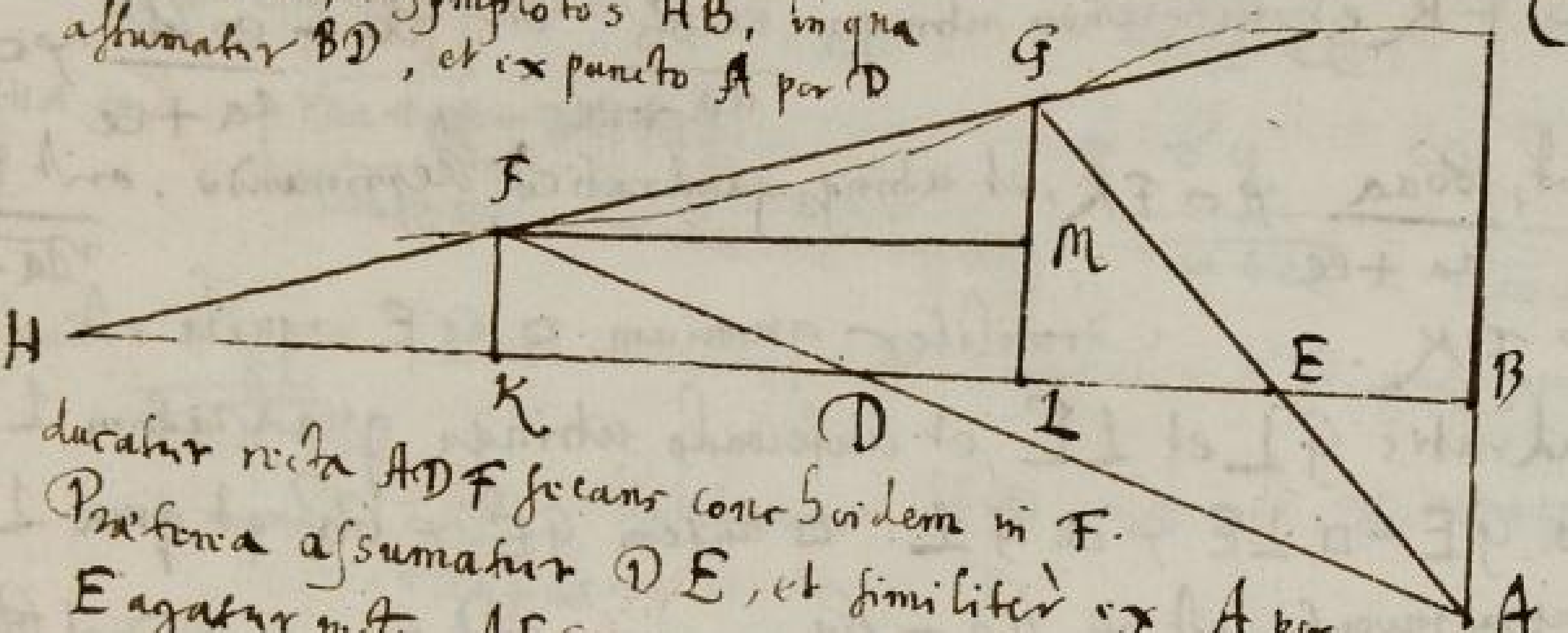
Quia $2 + \; = \sqrt{40044 - xx} - \sqrt{1 - xx}$.

De Linea Conchoide

1 Problema

Invenire rectam Tangentem Conchoidem in puncto dato F.

Sit Conchoides FGC, cuius polus A, centrum B, Asymptotos HB, in qua assumatur BD, et ex puncto A per D



ducatur recta ADF secans conchoidem in F.
Praeterea assumatur DE, et similiter ex A per E agatur recta AEG occurrens Conchoidi in G, ex G et F demittantur perpendiculares GL FK in HB, deinde iungantur F et G recta FG, quae protracta occurrat rectae HB in H. praesuppono autem ABC esse ipsi HB ad rectos.

Porro assumptis in hac constructione rectis, ut et investigandis dentur nomina, nempe.

$AB = a$

$BC.\ GE.\ FD = b$

$DB = e$

$DE = q$

$HD = x$

$\square DA = \square AB + \square DB$ id est $aa + ee$

Ergo $DA = \sqrt{aa + ee}$.

Quoniam autem propter similitudinem Triangulorum DBA et FKD, est ut DA ad DB, ita FD ad DK, id est

$\sqrt{aa + ee}$ ad e. ut b. ad $\dfrac{eb}{\sqrt{aa + ee}} = DK$.

Praeterea recta $DB - DE = EB$, quare $e - q = EB$, quare eius quadratum aequale est $ee - 2eq + qq$. $\square$ autem $EA = \square EB + \square AB$.

$ee - 2eq + qq + aa = \square EA$.

et $\sqrt{ee - 2eq + qq + aa} = EA$.

Propter similitudinem autem Triangulorum BEA et GEL erit ut EA ad EB sic EG ad LE, hoc est, ut $\sqrt{ee - 2eq + qq + aa}$ ad $e - q$ sic b ad $\dfrac{be - bq}{\sqrt{ee - 2eq + qq + aa}}$. LE ergo $= \dfrac{be - bq}{\sqrt{ee - 2eq + qq + aa}}$

$DE - LE = DL$ quare $q - \dfrac{be + bq}{\sqrt{ee - 2eq + qq + aa}} = DL$

Sed KD et DL sunt aequales ipsi KL. ergo $\dfrac{eb}{\sqrt{aa + ee}} + q - \dfrac{be + bq}{\sqrt{ee - 2eq + qq + aa}}$

aequatur KL.

Porro quoniam □ FD aequatur □ DK + □ FK, quadratum autem DK ex supra inventis est $\frac{eebb}{aa+ee}$ et □ FD $\propto bb$, erit ergo $bb \propto \frac{eebb}{aa+ee}$ + □ FK et auferendo utrinque $\frac{eebb}{aa+ee}$ erit $bb - \frac{eebb}{aa+ee} \propto$ □ FK id est, $\frac{bbaa}{aa+ee} \propto$ □ FK, et utrinque quadratice deprimendo, erit $\frac{ba}{\sqrt{aa+ee}} \propto$ FK. Similiter quoniam □ GE aequale est duobus quadratis GL et LE, et auferendo utrinque quadratum LE, erit □ GE − □ LE $\propto$ □ GL. □ autem GE $\propto bb$. et quia LE supra inventa est $\propto \frac{be-bq}{\sqrt{ee-2eq+qq+aa}}$ erit ergo □ LE $\propto \frac{bb \,|\, ee-2eq+qq}{ee-2eq+qq+aa}$

NB. Dum sic scribitur $\frac{bb \,|\, ee-2eq+qq}{ee-2eq+qq+aa}$ memineris multiplicandum esse bb in $ee-2eq+qq$, et productum dividi debere per $ee-2eq+qq+aa$

quare □ GE − □ LE $\propto \frac{bbaa}{ee-2eq+qq+aa}$. quare □ GL erit quoque $\propto \frac{bbaa}{ee-2eq+qq+aa}$ quare GL erit $\propto \frac{ba}{\sqrt{ee-2eq+qq+aa}}$

Jam vero in Schemate conchoidis ex puncto F demittatur perpendicularis FM in GL, tuncque erunt duo triangula GMF et FKH similia, quare erit, ut GM ad MF, ita FK ad KH. GM autem est aequalis GL − FK, quare GM erit aequalis $\frac{ba}{\sqrt{ee-2eq+qq+aa}} - \frac{ba}{\sqrt{aa+ee}}$

MF autem, id est KL ex supra inventis est aequalis $\frac{eb}{\sqrt{aa+ee}} + q - \frac{be+bq}{\sqrt{ee-2eq+qq+aa}}$ sive $\frac{eb}{\sqrt{aa+ee}} + \frac{bq-be}{\sqrt{ee+2eq+qq+aa}} + q$ ut et FK fuit inventa $\frac{ba}{\sqrt{aa+ee}}$ quare iam iuxta dictam

Ad pag. 3. linea 12 et sequent.

Limi $2 + q \propto \sqrt{40044 - xx} - \sqrt{1-xx}$.

proportionem erit: Ut $\frac{ba}{\sqrt{ee-2eq+qq+aa}} - \frac{ba}{\sqrt{aa+ee}}$ ad $\frac{eb}{\sqrt{aa+ee}} + \frac{bq-be}{\sqrt{ee-2eq+qq+aa}} + q$ ita $\frac{ba}{\sqrt{aa+ee}}$ ad KH

quae erit $\frac{q\sqrt{aa+ee-2eq+qq}}{\sqrt{aa+ee}-\sqrt{aa+ee-2eq+qq}} + \frac{bq}{\sqrt{aa+ee}-\sqrt{aa+ee-2eq+qq}} - \frac{be}{\sqrt{aa+ee}}$. Huic inventae KH, si addatur KD, quae supra inventa fuit aequalis $\frac{eb}{\sqrt{aa+ee}}$ habebimus HD, id est x. nempe omnibus ritè ordinatis inveniemus $\frac{q\sqrt{aa+ee-2eq+qq}+bq}{\sqrt{aa+ee}-\sqrt{aa+ee-2eq+qq}} \propto x$

Et omnibus per denominatorem multiplicatis erit $q\sqrt{aa+ee-2eq+qq}+bq \propto x\sqrt{aa+ee} - x\sqrt{aa+ee-2eq+qq}$. Addatur utrinque $x\sqrt{aa+ee-2eq+qq} - bq$ erit $q+x \text{ in } \sqrt{aa+ee-2eq+qq} \propto x\sqrt{aa+ee} - bq$. utraque aequationis pars si quadretur, habebimus $qqaa + 2aaqx + eeqq + 2eeqx - 2eq^3 - 4eqqx - 2eqxx + q^4 + 2xq^3 + qqxx \propto bbqq - 2bqx\sqrt{aa+ee}$. et omnibus per q divisis, erit $qaa + 2aax + eeq + 2eex - 2eqq - 4eqx - 2exx + q^3 + 2xqq + qxx \propto bbq - 2bx\sqrt{aa+ee}$.

Jam autem ex inspectione schematis patet, quo punctum E propius accedit ad punctum D, eo ipso punctum G quoque propius accessurum ad punctum F, adeo ut si punctum E coincidat cum puncto D, punctum quoque G incidat in F: et hoc in casu recta HFG a puncto H per F et G ducta tanget curvam in puncto F, quare ponemus punctum E coincidere cum puncto D, id est, magnitudinem ED (quae vocatur q) esse nullam, et ex proximè praecedenti aequatione tollamus omnes terminos, in quibus reperitur quantitas q, manifestum est nos tunc habituros aequationem exprimentem habitudinem ipsius x, id est rectae, HD, quae requiritur, ut recta HF a puncto H per datum punctum F ducta in eodem puncto F curvam tangat, quare erit

$$2aax + 2eex - 2exx \propto -2bx\sqrt{aa+ee}$$

& omnibus per $2x$ divisis erit

$$+aa+ee-ex \propto -b\sqrt{aa+ee}$$

addendo utrinque $b\sqrt{aa+ee} + ex$, habebimus

$$aa+ee+b\sqrt{aa+ee} \propto ex$$

Quare $\dfrac{aa+ee+b\sqrt{aa+ee}}{e} \propto x$. quae erat invenienda pro HD.

Hinc facilis oritur constructio pro inventione lineae determinantis tangentem conchoidis in quocunque puncto dato: nempe si per conchoidis CGF punctum quodcunque datum F velimus ducere rectam tangentem conchoidem in puncto F, oportet conjungere puncta F et A: atque ad DB. DA et FA invenire quartam proportionalem, quae sit HD, tunc si per puncta H et F ducatur recta HF, haec tanget conchoidem in puncto F dato, quod erat faciendum.

Obiter hic moneo. si multiplicer DA in FA erit ▭ DAFA, $aa+ee+b\sqrt{aa+ee}$ et hoc rectangulum si dividar per e hoc est per DB, proveniet x sive HD.

Haec ratio inveniendi Tangentem est expeditior, quam illa quae a Cartesio et Commentatore proponitur.

2 Problema.

In conchoide invenire confinia contrarii fluxus, sive quod idem est, invenire punctum flexus.

Sit conchoides HBFEG, cuius polus C, vertex B, asymptotos AD, oportet invenire confinia contrarii flexus. Pro fundamento autem investigationis subsecuturae sciendum est, si ab asymptoto puncto aliquo puta D ducta sit recta DE tangens curvam in puncto E, tunc etiam ab eodem puncto D duci posse alteram rectam puta DF tangentem conchoidem in alio quodam puncto at F existente in parte conchoidis con-

Ad pag. 3. linea 12 et sequenti

Quia $4 = \sqrt{4004\,4 - xx} - \sqrt{1 - xx}$.

trariè incurvata: Præterea notandum est (ductis rectis FC et EC secantibus rectam AD in punctis L et M) quo hæc puncta contactus E et F propius ad se mutuò accedunt, eo ipso quoque tangentem DE. DF. ut et rectas FC. EC. propius ad se mutuò accedere, et LM fieri minorem, donec tandem hæc puncta contactus E et F (existentia in utraque curvatura) coincidentia efficiant, ut tangentes DE DF ut et FC. EC. in unicam rectam coalescant, et LM nulla evadat, quo quidem in casu manifestum est tangentem transeuntem per hoc punctum coincidentiæ debere necessariò eandem tangere in communi termino contrarij fluxus.

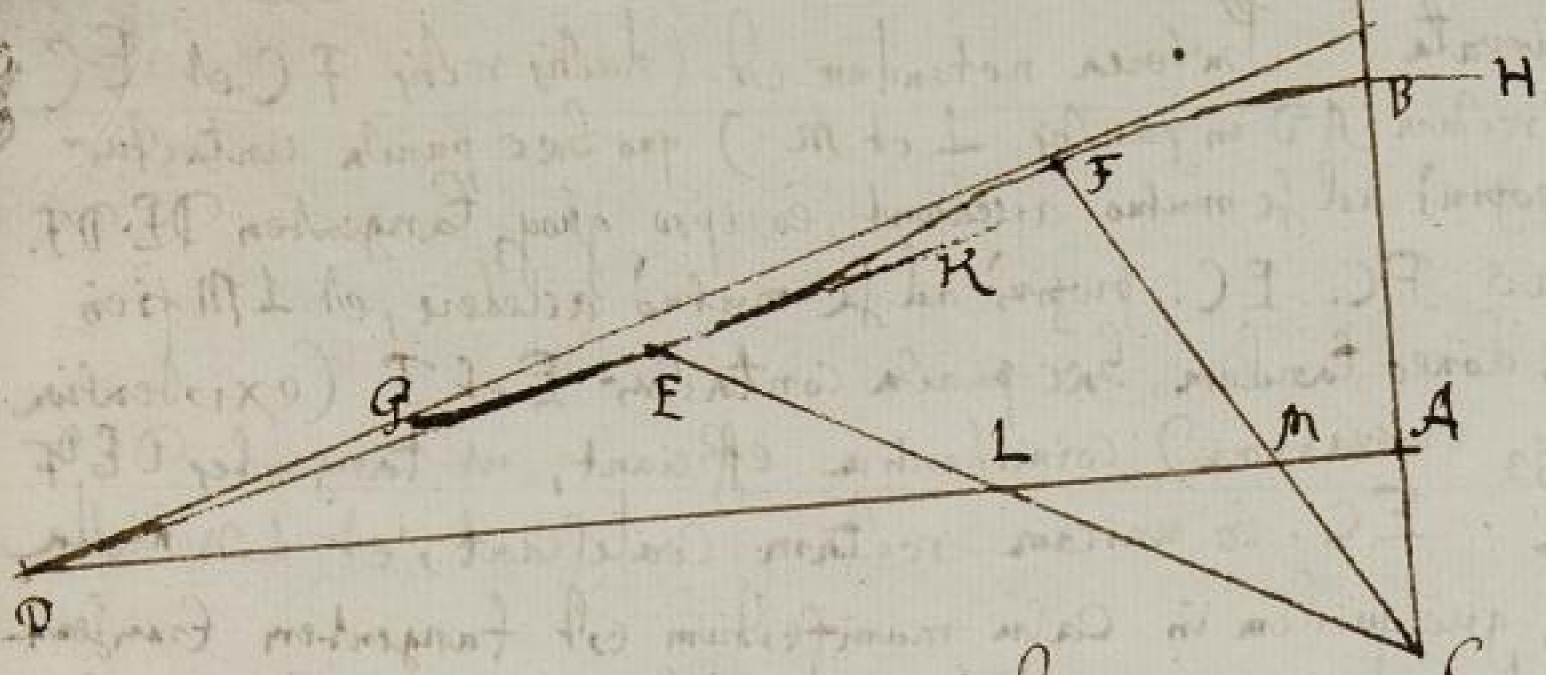
Sit itaque
AC $= a$
AB $= b$
DL $= 4$
LM $= y$
LA $= x$.

Erit itaque per præcedens problema $\dfrac{xx + aa + b\sqrt{aa + xx}}{x} = 4$

Explicationis causa addo hæc: Quia in præcedenti problemate demonstratum est esse ut LA ad LC, sic EC ad DL igitur fac, ut LA hoc est, ad LC $= \sqrt{\square\, LA + AC}$ sive $\sqrt{xx + aa}$: sic EC sive $b + \sqrt{xx + aa}$ ad DL $= 4$
$= \dfrac{xx + aa + b\sqrt{aa + xx}}{x}$.

Eodem modo per idem problema invenitur DM sive $4 + y = \dfrac{xx - 2xy + yy + aa + b\sqrt{aa + xx - 2xy + yy}}{x - y}$.

Explicationis causa. Quia per præcedens problema est, ut MA, hoc est $x - y$ ad MC sive $\sqrt{xx - 2yx + yy + aa}$ sic FC, hoc est FM + MC $= b + \sqrt{xx - 2yx + yy + aa}$ ad FC, quæ erit $\dfrac{xx - 2xy + yy + aa + b\sqrt{aa + xx - 2yx + yy}}{x - y}$



Auferatur prior aequatio ab hac posteriori, hoc est, subtrahatur DL de DM et restabit LM sive y

erit $\frac{xx-2xy+yy+aa+b\sqrt{aa+xx-2xy+yy}}{x-y} \quad \frac{-xx-aa-b\sqrt{aa+xx}}{x}$

aequalis y. Inveniatur communis denominator multiplicando $x-y$ per x. eritque ille $xx-yx$. per hunc multiplica posterius membrum aequationis, sive y. eritque illud $yxx-xyy$

Multiplica deinde prioris membri partem priorem per x, et posteriorem per $x+y$.

$$xx-2xy+yy+aa+b\sqrt{aa+xx-2xy+yy}$$
$$x$$

Prior pars primi membri $x^3-2xxy+yyx+aax+bx\sqrt{aa+xx-2xy+yy}$

Posterior pars primi membri

$$-xx-aa-b\sqrt{aa+xx}$$
$$+x-y$$
$$-y^3-aax-bx\sqrt{aa+xx}+xxy+aay+by\sqrt{aa+xx}$$

Iam redeatur ad aequationem, in qua nunc aequalia erunt

$$x^3-2xxy+yyx+aax+bx\sqrt{aa+xx-2xy+yy}-x^3-aax$$
$$-bx\sqrt{aa+xx}+xxy+aay+by\sqrt{aa+xx}$$

Ad pag. 3. linea 12 et sequent.

$\sqrt{4002\ldots - xx} - \sqrt{1-xx}$.

$\backsim yxx - xyy$

Deleantur delenda, et omnia transferantur ad alteram partem. erit

$$-2xxy + 2yyx + aay - bx\sqrt{aa+xx} + bx\sqrt{aa+xx-2xy+yy} + by\sqrt{aa+xx} \backsim 0$$

Deinde priorem quantitatem transferatis in alteram partem

$$bx\sqrt{aa+xx-2xy+yy} + by\sqrt{aa+xx} \backsim 2xxy - 2yyx - aay + bx\sqrt{aa+xx}$$

Utraque pars multiplicetur quadraticè.

$$\text{erunt } bbxxaa + bbx^4 - 2bbx^3y + 2bbxxyy + bbyyaa + 2bbxy\sqrt{\begin{matrix}aa+xx\\ aa+xx-2xy+yy\end{matrix}}$$

$$\backsim 4x^4yy - 8x^3y^3 + 4xxy^4 - 4xxyyaa + 4xy^3aa + a^4yy + bbxxaa + bbx^4 + 4bx^3y\sqrt{aa+xx} - 4byyxx\sqrt{aa+xx} - 2aabyx\sqrt{aa+xx}$$

Deleantur delenda, et reliqua dividantur per y

$$\text{erunt } -2bbx^3 + 2bbxxy + bbyaa + 2bbx\sqrt{\begin{matrix}aa+xx\\ aa+xx-2xy+yy\end{matrix}}$$

$$\backsim 4x^4y - 8x^3yy + 4xxy^3 - 4xxyaa + 4xyyaa + a^4y + 4bx^3\sqrt{aa+xx} - 4byxx\sqrt{aa+xx} - 2aabx\sqrt{aa+xx}.$$

Jam ex monitis preliminaribus patet, quando quantitas LM, id est y est nulla, tunc puncta contactus E et F, ut et FC et EC coincidentia ostendere terminum requisitum, quare putemus y esse aequalem nihilo, eamque ob causam deleamus in proxime praecedenti aequatione omnes terminos, in quibus y reperitur, tumque incidemus in aequationem sequentem

$$2bbaax \propto 4bx^3\sqrt{aa+xx} - 2aabx\sqrt{aa+xx}$$

Cum enim $-2bbx^3 + 2bbx\sqrt{aa+xx}$ neglectis caeteris quantitatibus, ubi occurrit $\frac{\sqrt{aa+xx}}{y}$ aequalis sit sequentibus $+4bx^3\sqrt{aa+xx} - 2aabx\sqrt{aa+xx}$. Si $+2bbx$ in $\frac{\sqrt{aa+xx}}{\sqrt{aa+xx}}$ hoc est, in $aa+xx$ multiplicaveris, erit productum $+2aabbx + 2bbx^3$. Atsic $-bbx^3$ atque $+2bbx^3$ se mutuo tollunt, adeoque recte ponitur $2bbax \propto 4bx^3\sqrt{aa+xx} - 2aabx\sqrt{aa+xx}$

Dividantur omnia per $2xb$.

erit $baa \propto 2xx\sqrt{aa+xx} - aa\sqrt{aa+xx}$

Utraque pars quadretur

eritque $bba^4 \propto +4x^6 + a^6 - 3a^4xx$

auferatur utrinque bba^4

et erit $+4x^6 - 3a^4xx + a^6 - bba^4 \propto 0$.

Haec aequatio habeatur pro cubica

Nam multiplicentur singula per 2

Erunt $8x^6 - 6xxa^4 + 2a^6 - 2bba^4 \propto 0$.

Sit vero jam $2xx \propto 4a$. sumendo a pro unitate

Multiplica cubice

Erit $8x^6 \propto 4^3a^3$

Et quia $2xx \propto 4a$, si multiplicas per 3

Erunt $6xx \propto 3\,4a$.

Ad pag. 3. linea 12 et sequenti

Multiplica per a^4

erunt $6xxa^4 = 3za^5$

Ergo $8x^6 - 6xxa^4 = z^3a^3 - 3za^5$

Adde $+2a^6 - 2bba^4$

Erunt $8x^6 - 6xxa^4 + 2a^6 - 2bba^4 = z^3a^3 - 3za^5 + 2a^6 - 2bba^4$

Sed retineatur posterior aequatio

$z^3a^3 - 3za^5 + 2a^6 - 2bba^4 = 0.$

Divide per a^3

$z^3 - 3zaa + 2a^3 - 2bba = 0.$

Pro sequenti constructione hac sequentia age.

Fiat, ut a ad b ita b ad y

erit $bb = ay$

Sit ergo $bb = ay$

et $2bb = 2ay$

et $2bba = 2aay$

$z^3 - 3zaa + 2aaa - 2aay = 0.$

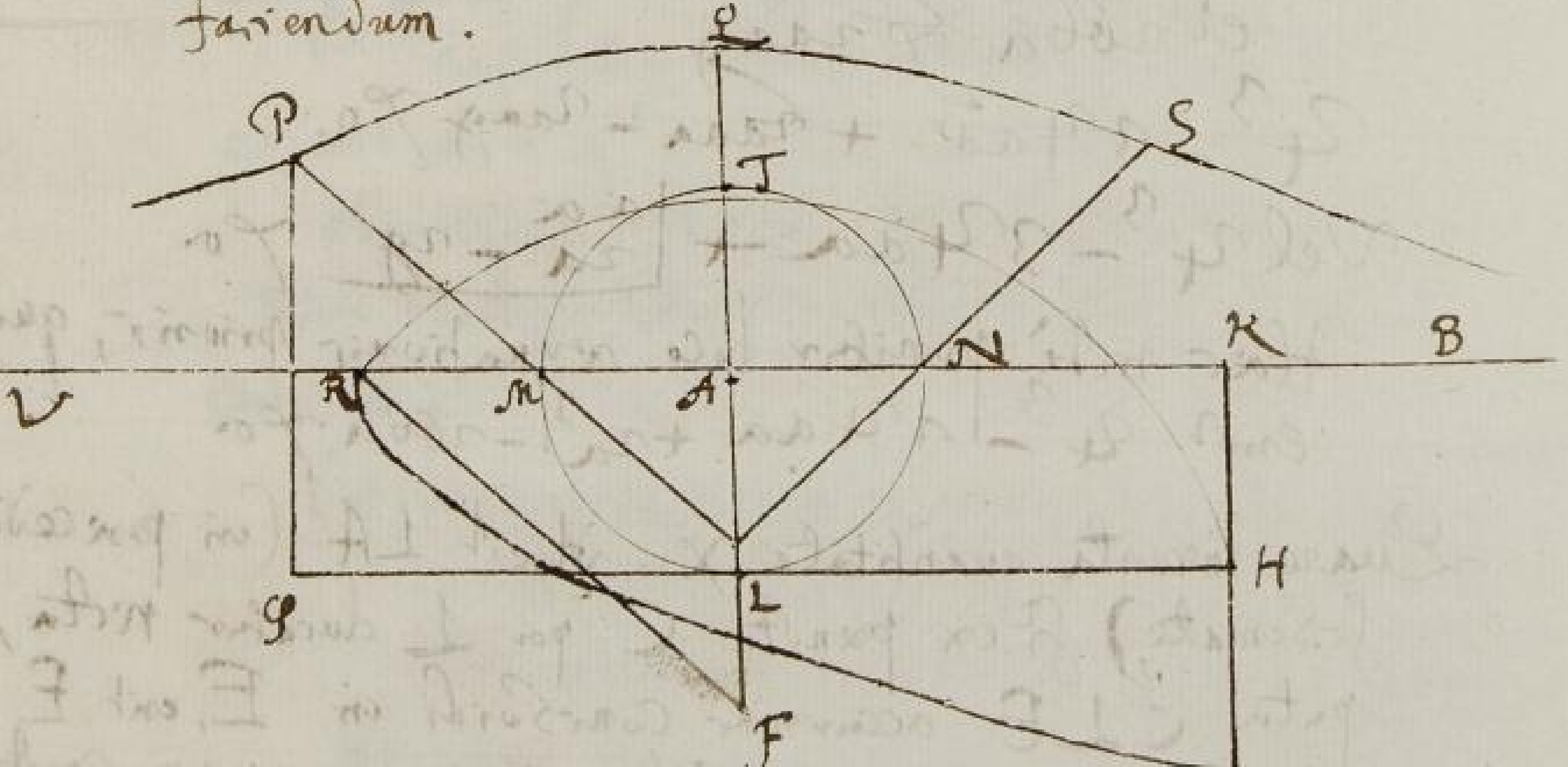
Vel $z^3 - 3zaa + aa\,\overline{2a - 2y} = 0$

Hac recte ponitur loco aequationis prioris, quae erat $z^3 - 3zaa + 2a^3 - 2bba = 0$

Quare inventa quantitate x, id est LA (in praecedenti schemate,) si ex puncto C per L ducatur recta, puta CLE occurrens conchoidi in E, erit E punctum quaesitum. Sed afferamus totam constructionem delineatoriam pro inventione puncti quaesiti

Respice igitur ad schema sequens.

Sit Conchoides PQS, cuius asymptotos BAV centrum A, polus G, vertex Q; fiatq TA aequalis GA, et ut GA ad AQ ita AQ ad FT, ponaturq AR dupla ipsius GA, deinde per methodum a Cartesio in sua Geometria traditam describatur Parabola RC, cuius latus rectum GA, vertex R, axis RAB. tum centro F radio FR describatur circulus secans parabolam in C, et ex C demittatur perpendicularis CK in axem RB, haec bisecetur in H, ex quo ducatur LH parallela ad AB, denique Diametro LT describatur circulus secans axem in M et N punctis, per quae, si ex foco G ducantur rectae GMP, et GNS secantes Conchoidem in P et S, erunt puncta P et S confinia contrarij flexus, quod erat faciendum.



Ad pag. 3. linea 12 et sequent.

Pro intelligenda constructione praemissa.

Puta AR huius Schematis esse illam, quae DA vocatur in Schemate Cartesiano. scilicet in Geometria Cartesiana. sed AD Cartesiana $\propto \frac{1}{2}a$ (hoc est AC) $+\frac{1}{2}p$, scilicet à C in D. et quia in nostro Schemate $p \propto 3$. sive $3a$ ergo $\frac{1}{2}p \propto \frac{3}{2}$ sed $\frac{a}{2}$ et $\frac{1}{2}a \propto 2$. hinc AR nostrum $\propto 2a$

Deinde tenendum $2xx \propto 4a$

si dividas per 2

erit $xx \propto \frac{1}{2}4a$

et $x \propto \sqrt{\frac{1}{2}4a}$

At AL $\propto \frac{1}{2}4$ et AT $\propto a$

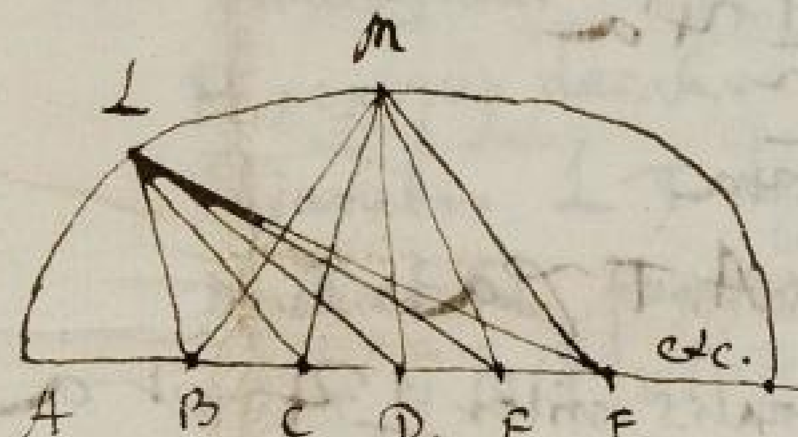
Ergo media proportionalis inter $\frac{1}{2}4$ et a hoc est, inter AL et AT $\propto x$. sive AM. Schematis penultimi.

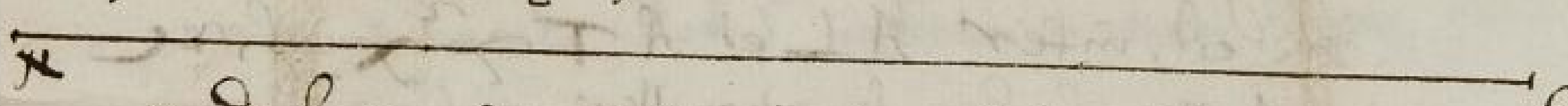
Constructio haec differt a Constructione Hugenii quia ille suam inventionem direxit ad perpendicularem ex puncto P super asymptoton AV demissam.

Quaestio.

ABCDEF is een rechte gegevene lyn, in welcke gegeven zynde 3. 4. 5. of meer punten nae wellgevallen, als A. K. G. B. C. D. E. etc. te vinden een kromme lyn, hebbende dese eygenschap, dat de Somme van alle de rechte

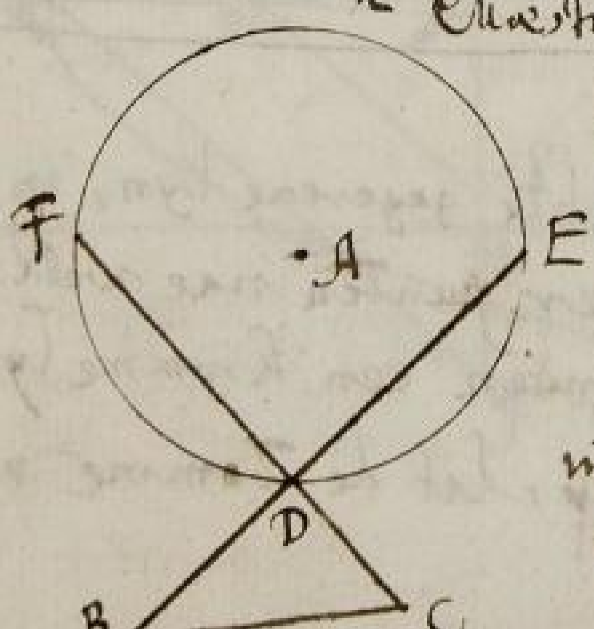
uyt yder punt der selve krummen lini, tot de gegeven punten
B. C. D. E. etc. gehaelt so groot zyn, als een gegeven
lini





De linien $BL + CL + DL + EL + FL$
ende $BM + CM + DM + EM + FM$ sullen der lini
XO gleich sein. ende sullen ooc $BL + CL + DL + EL + FL$
gleich sein den linien $BM + CM + DM + EM + FM$

~ Quaestio.



Dato circulo positione et magnitudine
atq extra circulum punctis B et
C. ex punctis B et C ducere
duas rectas, veluti CF et
BE ita ut spatia vel segmenta
intra circulum absumpta, veluti DF
et DE inter se sint aequalia.

Ad pag. 3. linea 12 et sequent.

Erit $z \propto \sqrt{\frac{400zz}{169} - xx} - \sqrt{1-xx}$.

Si pro x sumatur 0. vel nihil.

erit $z \propto \sqrt{\frac{400zz}{169}} - 1$

et $z+1 \propto \sqrt{\frac{400zz}{169}}$.

et $zz+2z+1 \propto \frac{400zz}{169}$.

et tandem $z \propto \frac{429}{231}$ ut habet Author.

Si x sit $\propto \frac{3}{5}$.

$z \propto \sqrt{\frac{400zz}{169} - \frac{9}{25}} - \sqrt{1-\frac{9}{25}}$. sed $\sqrt{1-\frac{9}{25}} \propto \sqrt{\frac{16}{25}}$ hoc est $\frac{4}{5}$.

Ergo $z+\frac{4}{5} \propto \sqrt{\frac{400zz}{169} - \frac{9}{25}}$

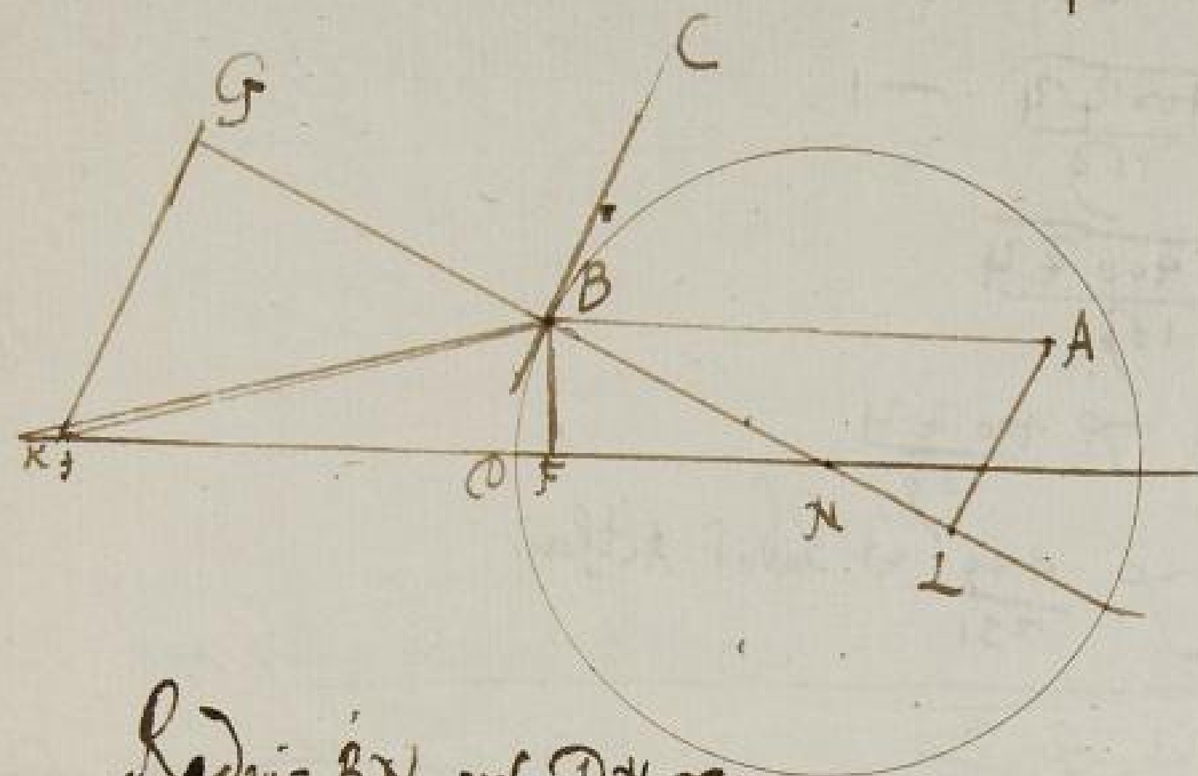
et tandem $z \propto \frac{1873}{5,231}$ ferè.

Ad pag. 4. lin. 13.

KF sive KN + NF. esse ad FB, ut KI + alia quaedam linea ad IM + alia quad. linea.

Per KI + aliam quad. lineam intelligit Author, numerum exprimentem KI, non ipsam KI exhibere, sed quantitate ipsa quantitate KI paulo maiorem: excessum vero ipsum incognitam longitudinem appellat + aliam lineam. Sic numerus $\frac{272}{5,231}$ cum sit maior quam KI, vocatur KI + alia quaedam linea: et sic ipse intelligit de reliquis.

Ad p. 7. circa finem.



Radius BN vel DN $\propto 1$.

BF $\propto x$

IB $\propto$ AB

ID $\propto z$

AL $\propto 13 \propto y$

IG $\propto 20$.

ut BF ad BN sic AL ad AB sive BI

$x. \quad 1. \quad y. \quad \frac{y}{x} \propto$ BI.

Ergo a IB $\propto \frac{yy}{xx}$ unde subtracto quadr. BF $\propto xx$ relinquitur a IF $\propto \frac{yy}{xx} - xx$

et ideo IF $\propto \sqrt{\frac{yy}{xx} - xx}$

Porro subtracto DF $\propto 1 - \sqrt{1-xx}$ de IF $\propto \sqrt{\frac{yy}{xx} - xx}$

et relinquetur ID $\propto \sqrt{\frac{yy}{xx} - xx} - 1 + \sqrt{1-xx}$.

Amplius quia est BN ad BF ut IN ad IG.

hoc est 1 ad x ut $z+1$ ad $xz + x$

16 erit IG $\propto xz + x$

ut vero 13 ad 20 sic est y vel LA ad IG $\propto xz + x$

et ideo $13xz + 13x \propto 20y$

et $\frac{13xz + 13x}{20} \propto y$.

et a $\frac{169xxzz + 2,169xxz + 169xx}{400} \propto yy$

sed qu
meno

Fac re-
pleatur
aqua

Sed quia z vel FD superius erat $\sqrt{\frac{yy}{xx} - xx} - 1 + \sqrt{1-xx}$ ergo substituendo valorem ipsius yy, erit

$$z \text{ erit } \propto \sqrt{\frac{169zz + 2{,}169z + 169}{400} - xx} \; - 1 + \sqrt{1-xx}$$

Sed jam invenienda z dum $x \propto$ nihilo.

In hoc casu $z \propto \sqrt{\frac{169zz + 2{,}169z + 169}{400}}$

et tandem $z \propto \frac{429}{231}$

Oporteat jam invenire z dum $x \propto \frac{7}{25}$

Cum enim $z \propto$ erat $\sqrt{\frac{169zz + 2{,}169z + 169}{400} - xx} \; - 1 + \sqrt{1-xx}$

Substitutis $\frac{7}{25}$ pro x et $\frac{49}{625}$ pro xx

erit $z \propto \sqrt{\frac{169zz + 338z + 169}{400} - \frac{49}{625}} \; - \frac{1}{25}$.

et $z + \frac{1}{25} \propto \sqrt{\frac{169zz + 338z + 169}{400} - \frac{49}{625}}$

et tandem $z \propto \frac{9691}{5775}$.

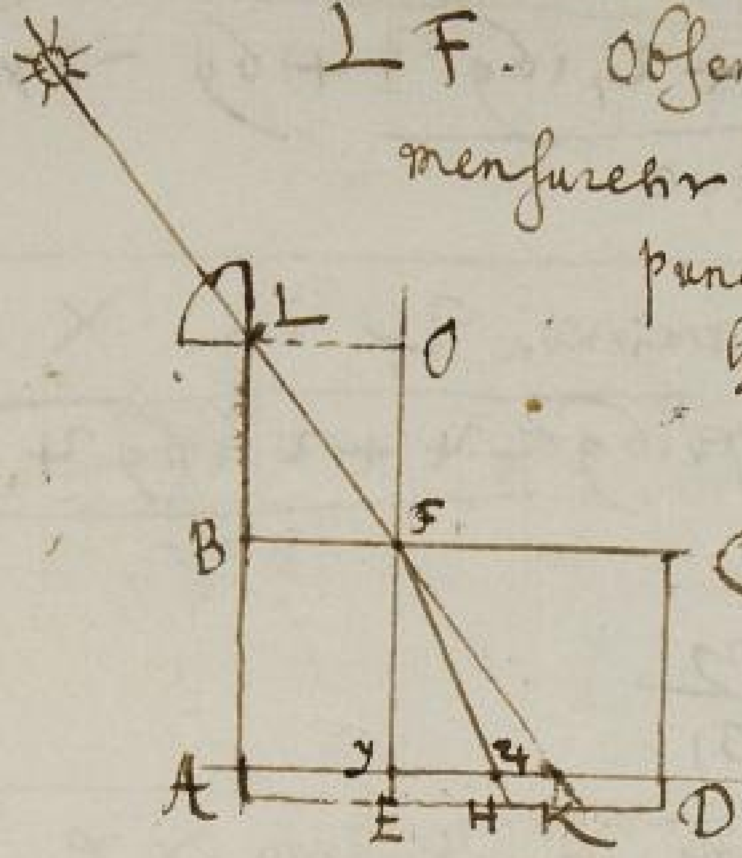
Quandoquidem vero Author pag. 3 l. 10 meminit, rationem linearum AL et GI esse ut 20 et 13, igitur operae pretium me facturum credo, si exposuero, quo artificio rationis dicti numeri investigari queant.

17

Sumatur vas ABCD forma parallelepipedae, sitque seu ligneum, seu lapideum, seu cupreum, seu aeneum F: (vel sumatur parallelepipedum ex vitro, si rationem inter radium incidentem in aere et refractum in vitro inquirere est animus.

F quod repleatur aqua

et sinus lateri AB continuato applicatur quadrans in L
Luceat jam Sol, et eius radius in superficiem BC incidens sit
LF. Observetur ergo punctum F. hoc est,
mensuretur distantia FB. observetur et
punctum imaginis Solaris in H,
hoc est, mensuretur intervallum
AH. Subtrahatur BF sive
AE de AH et restabit EH.
Mensuretur et BA, quae aequalis est lineae EF. Mensuretur denique et LF.



Deinde in triangulo rectangulo EFH investigetur longitudo lineae FH. Sumatur F4 aequalis ipsi FL et ducatur 4y parallela lineae EH. ac ponatur in Reg. proportion. ut FH ad HE sic F4 (sive LF) ad 4y.

~~Si capiatur BF inven~~

Si igitur BF inventa sit 20 partium et 4y 15. dicas rationem radij incidentis ex aere ad radium refractum in aqua mensuratam in perpendicularibus distantijs LO et 4y esse ut 20 ad 15.

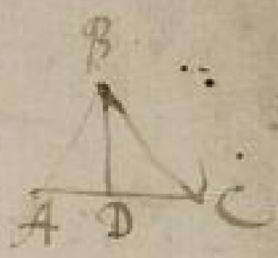
Et tali artificio deprehendes, si ABCD sit corpus vitreum, LO ad 4y sese habere ut 3 ad 2 ~~fere~~ sive ut 20 ad 13.

Cartesius rationem hanc docet invenire inter aerem et vitrum. in Gallica sua Dioptrica p. 137. Discours dixiesme

18

Invenire duo Triangula aequicrura, ejusdem areae, ejusdemque ambitus, et quorum singula latera et perpendicula sint ut numerus ad numerum.

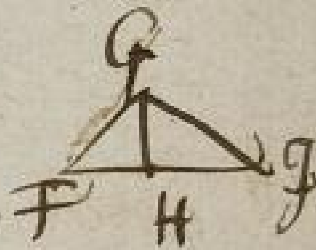
1. Quia latera debent absolutis numeris exprimi pro latere AB pono $aa + bb$ et pro AD, $2ab$, unde DB futurum est $aa - bb$, qui si quadrato $2ab$ subtrahatur de quadrato $aa + bb$ relinquitur quadratum $aa - bb$



$$\begin{array}{ll} & aa + bb \\ \square & a^4 + 2aabb + b^4 \\ \text{Subtrahe.} & +4aabb \\ \hline \text{Restat.} & a^4 - 2aabb + b^4 \end{array} \qquad \begin{array}{l} aa - bb \\ \hline \square \cdot a^4 - 2aabb + b^4 \end{array} \qquad \begin{array}{l} 2ab \\ 2ab \\ \hline +4aabb \end{array}$$

2. Deinde quia latera Triang. rectang. ABD, nimirum AB et AD constituta sunt $aa + bb$ et $2ab$ igitur ambitus Trianguli aequicruri ABC est $2aa + 2bb + 4ab$. quem invenies duplicando latera AB, et AD

3. Jam porro in secundo triangulo FGJ, si eodem modo pro FG ponatur $kk + dd$, et pro FH latere $2kd$, et pro GH latere $kk - dd$ erit ambitus Δguli FGJ Y. $2kk + 2dd + 4kd$.



Quia vero ambitus prior $2aa + 2bb + 4ab$ aequalis esse debet ambitui huic: $2kk + 2dd + 4kd$, igitur exinde patet $k + d$ aequari debere ipsi $a + b$.

4. Ac proinde in secundo Triangulo, quia quantitates k et d ignorantur, ponatur pro k, litera $a + x$, et pro d, statuatur $b - x$., et sic inveniantur latera FH, et HG. Et FH quidem innotescit, si sumas valorem $2kd$. Duc igitur k in d bis, hoc est, duc $a + x$ in $b - x$ bis.

$$\begin{array}{r} a + x \\ b - x \\ \hline ab + bx \\ -ax - xx \\ \hline \end{array}$$

Erit kd Y. $ab + bx - ax - xx$

Et duplicando quantitates hasce

Erit $2kd$ Y. $2ab + 2bx - 2ax - 2xx$.

At pro perpendiculari GH jam antea depictata erat $kk - dd$

Ergo pro k ponendo iterum $a + x$ et pro d sumendo $b - x$. 19

~~Erit kk~~

et [illegible] lateri AB continuato applicatur quadratum in L
Luceat jam Sol et eius radii [illegible]

erit $FH - ll = aa - bb + 2ax + 2bx$.

Atq ita tales terminos habent lineæ

FH quidem $= 2ab - 2ax + 2bx - 2xx$

GH autem $= aa - bb + 2ax + 2bx$.

5°. Due FH in GH, et acquires areã Trianguli posterioris.

$$FH = 2ab - 2ax + 2bx - 2xx$$
$$HG = aa - bb + 2ax + 2bx.$$

Multiplicatio.

$$2a^3b - 2a^3x + 2aabx - 2aaxx$$
$$-2ab^3 + 2abbx - 2b^3x + 2bbxx.$$
$$+ 4aabx - 4aaxx + 4abxx$$
$$-4ax^3 + 4abbx - 4abxx + 4bbxx - 4bx^3.$$

Atq hæ quãtitates sunt area Trianguli posterioris. Cum ergo prioris area sit $2a^3b - 2ab^3$ erit, inter hasce quãtitates æquatio.

Adde utrinq $4ax^3$, et $4bx^3$ quia x^3 est potestas summa ignotæ quantitatis

Erit $+2a^3b - 2a^3x + 6aabx - 6aaxx - 2ab^3 + 6abbx - 2b^3x + 6bbxx$

$= +2a^3b - 2ab^3 + 4ax^3 + 4bx^3.$

Et addendo utrinq $2ab^3$ ac tollendo $2a^3b$.

$+6aabx - 2a^3x - 6aaxx + 6abbx - 2b^3x + 6bbxx$

$= +4ax^3 + 4bx^3.$

Sive ut potestatis incognitæ x^3 termini maneant affirmati, et tota comparetur cum nihilo.

Erit $+4ax^3 + 4bx^3 - 6aabx + 2a^3x + 6aaxx - 6abbx + 2b^3x - 6bbxx = 0.$

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Quod totum dividi potest per $4ax + 4bx$ et manet.

$$xx + \frac{3}{2}ax - \frac{3}{2}bx + \frac{1}{2}aa - 2ab + \frac{1}{2}bb = 0.$$

Sive

$$xx + \frac{3}{2}ax - \frac{3}{2}bx = -\frac{1}{2}aa + 2ab - \frac{1}{2}bb.$$

vel quod idem est.

$$xx + \frac{3ax - 3bx}{2} = \frac{+4ab - aa - bb}{2}.$$

Iam ut habeatur x. addo quadratum semissis coefficientis, id est $\frac{3a-3b}{4}$ ad quantitatem post signum $=$ positam, nimirum ad $\frac{+4ab-aa-bb}{2}$

Sumatur ergo primo quadratum de $\frac{3a-3b}{4}$.

Eritque illud $\frac{9aa-18ab+9bb}{16}$.

Huic ut addi queat $\frac{+4ab-aa-bb}{2}$ redigatur prius ad eandem denominationem,

Eritque $\frac{9aa-18ab+9bb}{16}$ et $\frac{-8aa+32ab-8bb}{16}$.

unde summa erit $\frac{+aa+14ab+bb}{16}$.

Ab huius quantitatis $\frac{+aa+14ab+bb}{16}$ radice subtrahe semissem coefficientis, et habebis x.

$$x = \frac{-3a+3b}{4} + \sqrt{\frac{aa+14ab+bb}{16}}.$$

8vo. Quia vero x numerus rationalis esse debet, necesse est, ut quoque $aa+14ab+bb$ reiecto denominatore sit numerus quadratus, quocirca ut hoc obtineamus, ponimus et huius lateri AB continuato applicatur quadrati in L

Luceat jam Sol, et eius radius in ...

radicem ejusque $a+b+c$. ubi c et b cognitae praesupponuntur, a vero ignoratur.

Erit proinde aequatio talis.

$$aa+2ab+2ac+bb+2bc+cc \propto aa+14ab+bb$$

Et $2ac+2bc+cc \propto 12ab$

sive

$+2bc+cc \propto 12ab-2ac$

Et applicando singula ad quantitates in a ductas.

Et $\frac{+2bc+cc}{12b-2c} \propto a$.

Sumatur ergo quilibet numerus pro b, et quilibet pro c, ita tamen, ut a maior proveniat, quam assumpta b fuerit, quia superius erat $aa-bb$.

Quare si $b \propto$ fit 1. et $c \propto 3$. habebo

$a \propto \frac{5}{2}$

Respice n. ad inventam aequationem $\frac{+2bc+cc}{12b-2c} \propto a$.

Haec est $\frac{6+9}{12-6}$ sive $\frac{15}{6} \,\Big|\, \frac{5}{2}$. $\propto a$

Ut vero clarius x respiciendum est ad aequationem superius traditam. $x \propto -\frac{3a}{4}+\frac{3}{4}b+\sqrt{\frac{aa+14ab+bb}{16}}$.

$aa+14ab+bb \propto \frac{169}{4}$ et $\frac{aa+14ab+bb}{16} \propto \frac{169}{64}$.

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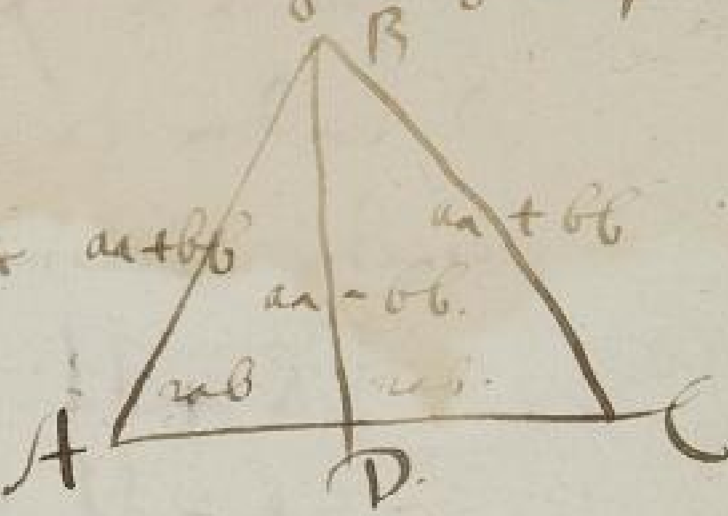
unde ~~radix~~ extractum erit $\frac{13}{8}$.

Deinde inventi huius radici $\frac{13}{8}$ adde $\frac{3b}{4}$. erit summa $\frac{19}{8}$.

Tantô de $\frac{17}{8}$ subtracto $\frac{3a}{4}$ sive $\frac{15}{8}$ remanebit $\frac{1}{8}$ sive $\frac{1}{2}$ pro x.

7 Ut vero ultimo tandem exhibeantur triangula optata, exprimendi sunt valores assumptarum literarum.

In primo Δ. erat aa+bb sic.



$a \propto \frac{5}{2}$ Ergo $aa \propto \frac{25}{4}$.

$b \propto 1.$ et $bb \propto 1.$

$ab \propto \frac{5}{2}$ et $2ab \propto \frac{10}{2}$.

Erit ergo $aa+bb \propto \frac{25}{4} + \frac{4}{4}$. sive rejecto denominatore 29.

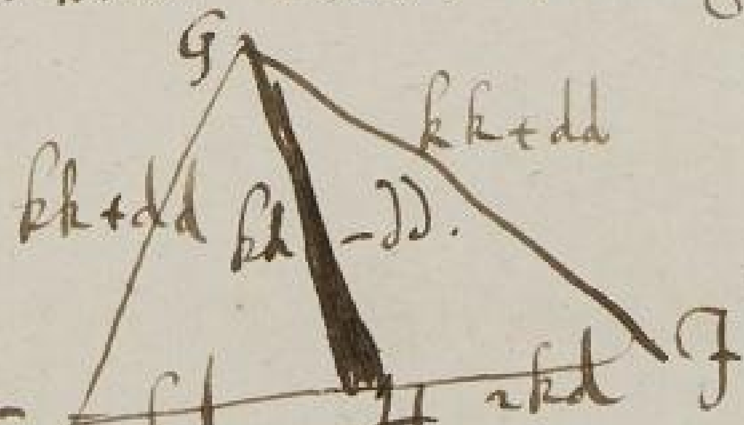
Et $aa-bb \propto \frac{25}{4} - \frac{4}{4}$ sive $\frac{21}{4}$. vel 21. rejecto quoque denominatore.

Et $2ab \propto \frac{20}{4}$. vel 20.

Proinde $AB \propto 29$. AD. 20. et $BD \propto 21$.

Ambitus huius triang. est $\begin{matrix}29\\29\\20\\\hline 98.\end{matrix}$ Area vero 420.

In posteriori autem Triangulo



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$k \propto a+x$ sive $\frac{5}{2}+\frac{1}{2}$ hoc est 3.
$d \propto b-x$ hoc est $\frac{2}{2}-\frac{1}{2}$ sive $\frac{1}{2}$. $kk \propto 9.$ $dd \propto \frac{1}{4}.$

et huius lateri AB continuato applicatur quadratum in L

At $kk + dd \propto 9 + \frac{1}{4}$ sive $\frac{37}{4}$ reducendo scilicet 9 sub denominationem eandem cum fractione $\frac{1}{4}$.

Ergo $kk - dd$ est $\frac{35}{4}$.

Et quia $2k \propto 6$. Ergo $2kd \propto \frac{6}{2}$ ac reducendo ad eandem cum 4 denominatione $\frac{12}{4}$ et sublatis communibus denominatoribus.

$kk + dd \propto 37.$

$kk - dd \propto 35.$

$2kd \propto 12.$

Unde ambitus Δ erit 98 et area similiter ut antea 420.

Proinde inventa sunt 2 triangula [illegible] quod faciendum erat.

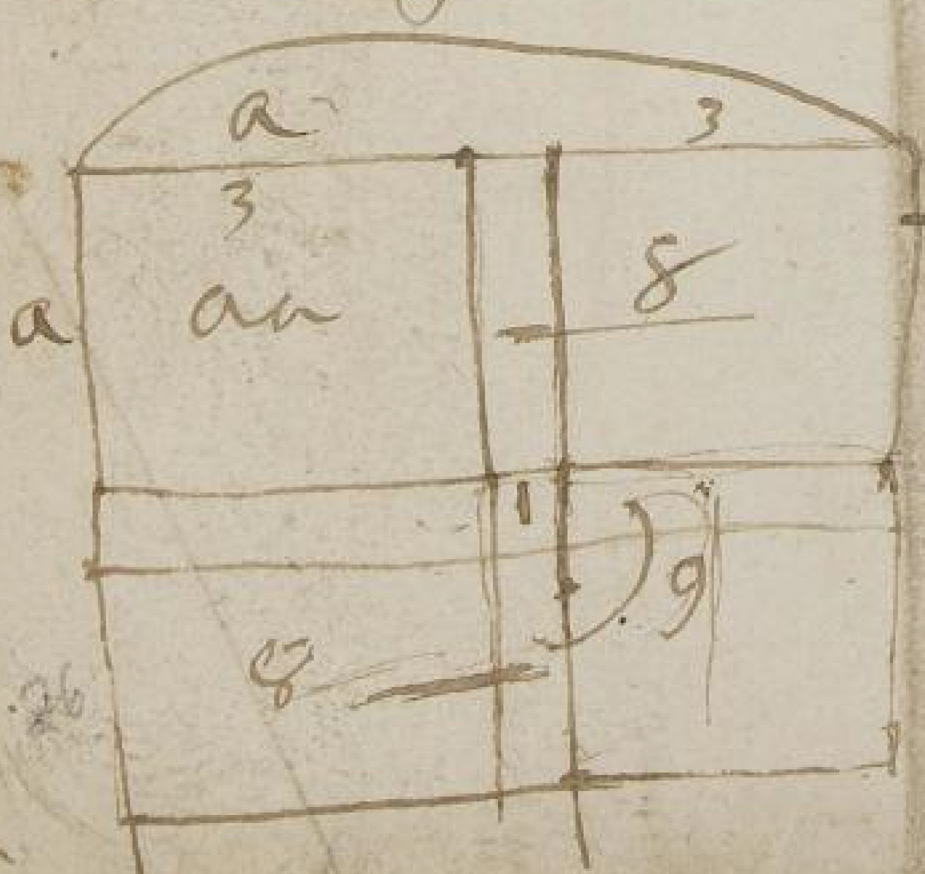
et huius lateri AB continuato applicatur quadratum in L

$\sqrt{ab+cd+fg}$

$\sqrt{ab+\sqrt{cd}+\sqrt{fg}}$

$\sqrt{ab+\sqrt{cd+\sqrt{fg+st+\ldots}}}$

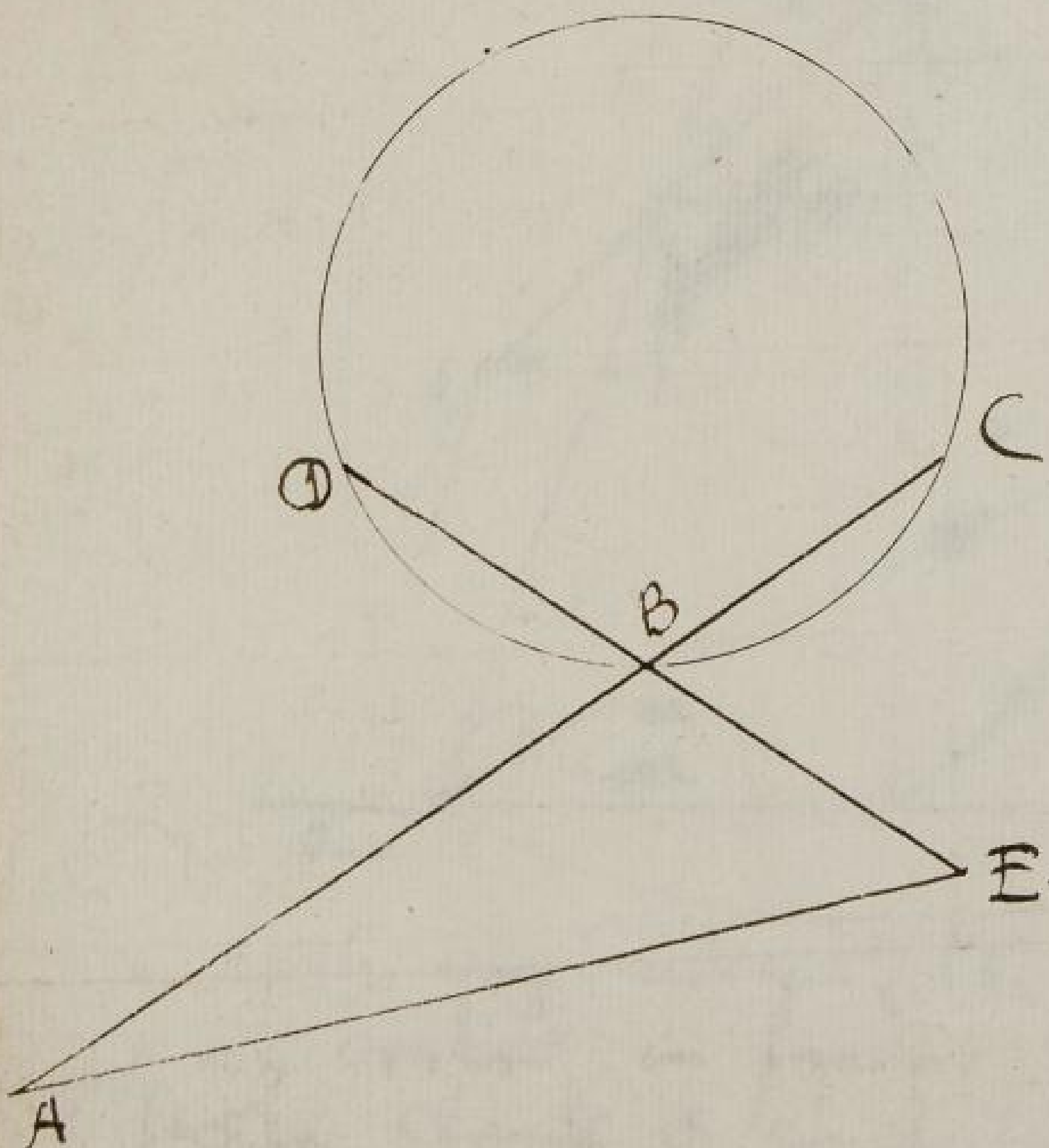
$aa + ab = cc$



$-\frac{1}{2}b + \sqrt{\frac{1}{4}bb + cc}$

Diese folgende Quaestio ist vor etlicher Zeit allhier den liebhabern der Algebra proponiret worden, welche prima fronte scheinet leichte zu sein, hatt aber grosse difficultät, denn erlange ich eine Aequationem quadrato-quadraticam affectam, welche nicht anders als per Geometriam Solidorum zu resolviren, es sey dann dieselbe per Resolutionem potestatum Vietaeam in numeris erörtert werde.

Ex datis punctis A et E ducere duas lineas veluti AC et ED quae intra circulum positione et magnitudine datum, absumant duo aequalia segmenta BD et BC.



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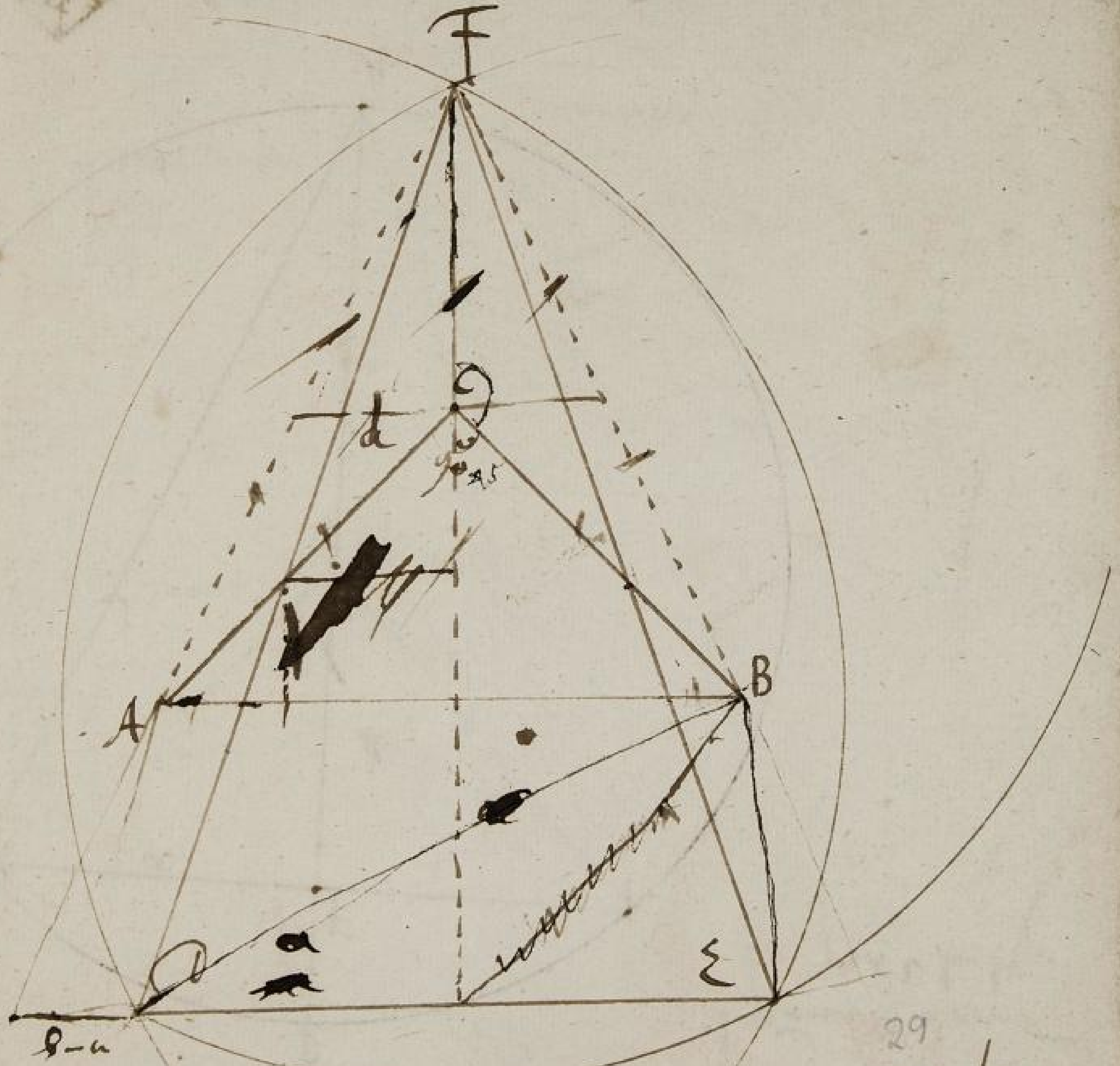
9
9 1 = 10



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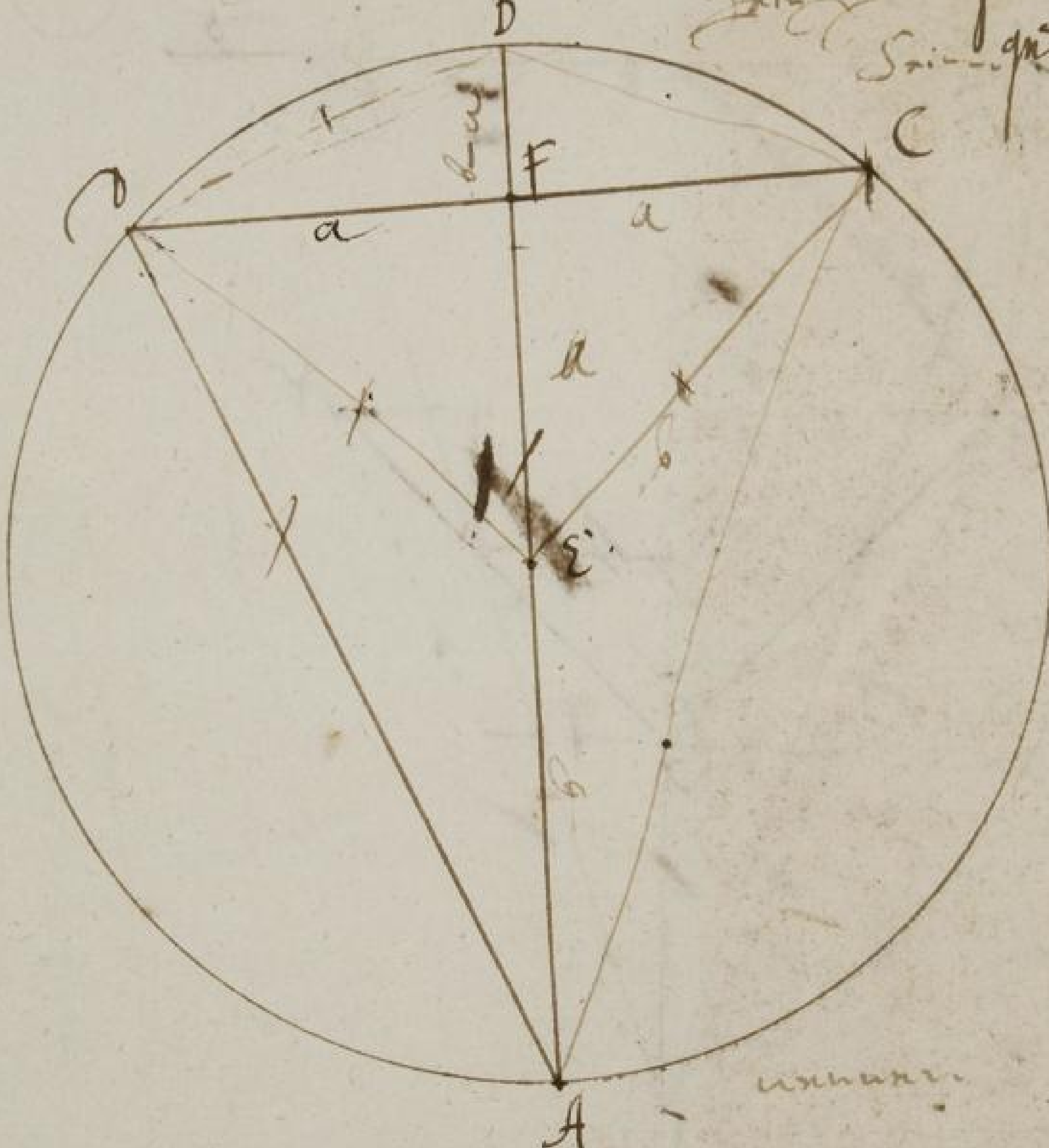
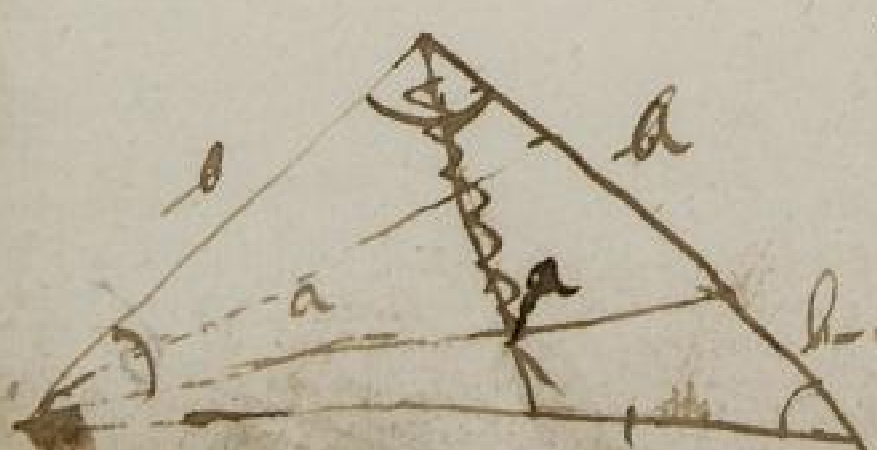
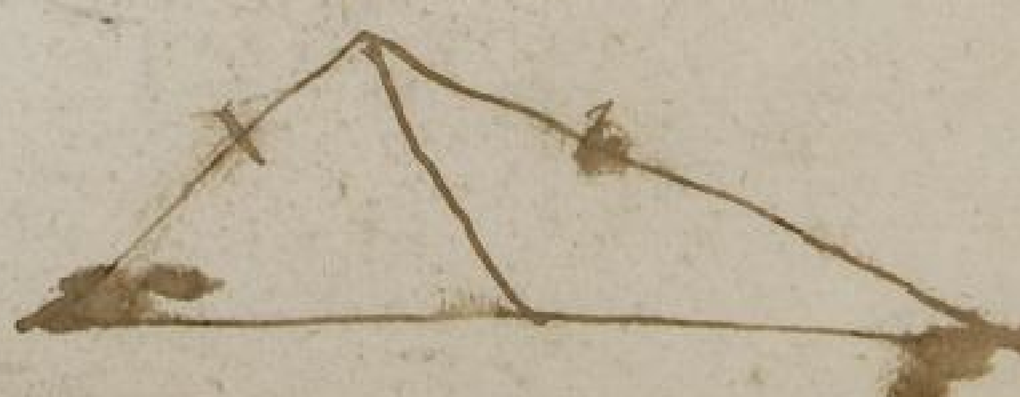
Ex Apicibus trianguli rectanguli aequicruri ABC. eodem radio descripti sint arcus circulorum, in punctis D E F sese intersecantes. ~~[illegible]~~ et ducti sint lineae DF, FE ED. Dabis autem latera Triang. ABC una cum radio BF, vel BD vel BE. quaeris latera Anguli DFE.

AB. & DA dantur
determinatæ & cognitæ
mensuræ. FC autem
æquatur FE, sed
quærimus earum
quantitatem

Monsieur Monsieur



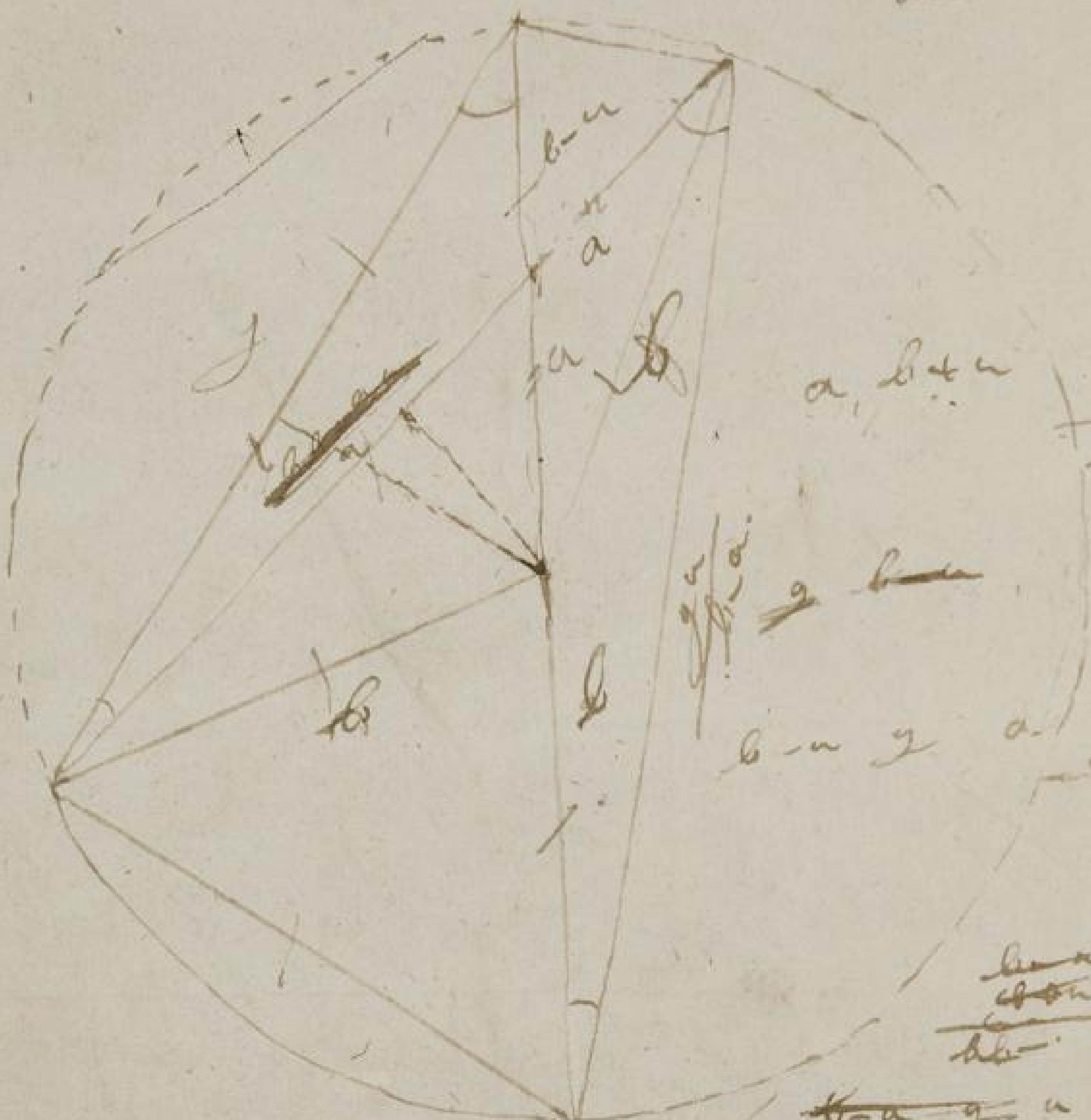




$9bb - 9aa = bba - 2baa + aaa$

$\cancel{9bb} = aaa + \overset{-2b}{9}aa + bba = 9bb$

$aa + 7a = 6050$



$a, \; b+a$

$b-a$
$b+a$
$bb + ba - ba$
$- aa$

$\frac{bb-aa}{a}$

$9 \quad \cancel{b-a}$

$b-a \quad 9 \quad a \quad \frac{9a}{b-a}$

$\cancel{b-a}$
$\cancel{b+a}$
$bb-$

$\cancel{b-a} \quad 9 \quad a$

$\frac{9a}{}$

$\frac{9a}{b-a} \quad a \quad b-a$

$\frac{ba-aa}{p} \quad \frac{9a}{b-a}$

$\frac{bbq - 2bap + aaq}{9}$

$ba - aa$
$b - a$
$bba - baa$
$- baa + aaa$

et lineis lateri AB continuato [illegible]

32

et huius lateri AB continuato ...

34

et sinus lateri AB continuato [illegible]