$i = v\cos\varrho - t\sin\varrho + \varepsilon$, $k = v\sin\varrho + t\cos\varrho + \vartheta$, ubi ε et ϑ sunt coordinatae puncti initialis systematis novi ad prius systema relatae, ϱ vero angulus inter axes abscissarum k et t interceptus. Facta hac substitutione prodit aequatio haec:

$$(r^2\cos\varrho^2 + s^2\sin\varrho^2)\, v^2 - 2\,(r^2 - s^2)\,\sin\varrho\cos\varrho.\, vt + (r^2\sin\varrho^2 + s^2\cos\varrho^2)\, t^2 \qquad (Q)$$
$$+ 2\,(r^2\varepsilon\cos\varrho + s^2\vartheta\sin\varrho)\, v - 2\,(r^2\varepsilon\sin\varrho - s^2\vartheta\cos\varrho)\, t + r^2\varepsilon^2 + s^2\vartheta^2 - s^2 b^2 = 0$$

Ergo universalis aequatio (U) tum tantum significabit ellipsin aliquam, si fieri potest, ut ex aequationibus, quas suppeditant collati inter se coefficientes aequationum (Q) et (U), eruantur valores reales quantitatum r, s, ϱ, ε, et ϑ. Coniunctis vero primum coefficientibus trium priorum utriusque aequationis terminorum facile reperiuntur haec:

$$A + C = r^2 + s^2,\ 4AC = (r^2 - s^2)\sin 2\varrho^2 + 4r^2s^2,\ B^2 = (r^2 - s^2)\sin 2\varrho^2,$$

itaque I. $4r^2s^2 = 4AC - B^2$, $(r^2 - s^2)^2 = (A - C)^2 + B^2$,

$$\text{II. } r^2 = \tfrac{1}{2}(A + C) + \tfrac{1}{2}\sqrt{[(A-C)^2 + B^2]},\quad \text{III. } s^2 = \tfrac{1}{2}(A + C) - \tfrac{1}{2}\sqrt{[(A-C)^2 + B^2]},$$

$$\text{IV. } \frac{r}{s} = \frac{\sqrt{(4AC - B^2)}}{A + C - \sqrt{((A-C)^2 + B^2)}}$$

$$(r^2 - s^2)\sin 2\varrho = \sin 2\varrho\sqrt{[(A - C^2) + B^2]},\ = -B,\ \text{ergo V. } \sin 2\varrho = -\frac{B}{\sqrt{[(A-C)^2 + B^2]}}$$

$$\sin\varrho = \sqrt{\left(\tfrac{1}{2} - \frac{A - C}{2\sqrt{((A - C^2 + B^2)}}\right)},\ \cos\varrho = \sqrt{\left(\tfrac{1}{2} + \frac{A - C}{2\sqrt{((A-C)^2 + B^2)}}\right)}$$

Collatis reliquorum terminorum coefficientibus prodeunt haec:

$$\text{VI. } \varepsilon = \frac{D\cos\varrho - E\sin\varrho}{2r^2},\quad \text{VII. } \vartheta = \frac{D\sin\varrho + E\cos\varrho}{2s^2}$$

$$\text{VIII. } b^2 = \frac{AE^2 + CD^2 - BDE - F(4AC - B^2)}{(4AC - B^2)\ [\tfrac{1}{2}(A+C) + \tfrac{1}{2}\sqrt{[(A-C)^2 + B^2)]}}$$

$$\text{IX. } a = \frac{AE^2 + CD^2 - BDE - F(4AC - B^2)}{(4AC - B^2)\ [\tfrac{1}{2}(A+C) - \tfrac{1}{2}\sqrt{[(A-C)^2 + B^2)]}}$$

Primum ex aequationibus II - IV cognoscitur, curvam aequatione (U) expressam tum tantum esse ellipsin, ubi fuerit $4AC > B^2$ (cf. §. 14): nam sub ea tantum conditione fieri potest, ut ellipseos axium ratio inveniatur non imaginaria. Ipsa axium quantitas definitur aequationibus VIII et IX, positio vero eorum formulis V - VII; nam punctum O Fig. 2, in quo abscissarum t axis SS′ secatur ellipseos axe FL, determinatur aequatione hac:

$$\text{X. } D'O = \frac{\varepsilon}{\sin\varrho} = \frac{D\,\text{cotg}\,\varrho - E}{2r^2}$$

Denique centri C′ coordinatae $CK = v''$ et $DK = t''$ etiam facile reperiuntur; quum enim sit $OC = OQ + CQ = OD\cos\varrho + \vartheta,\ C'K = -OC\sin\varrho,\ D'K = OD' - OK = OD' - OC'\cos\varrho$, ex his et praecedentibus erui possunt valores hi:

$$\text{XI. } v'' = \frac{B'E - 2CD}{4AC - B^2},\quad \text{XII. } t'' = \frac{BD - 2AE}{4AC - B^2}$$

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