One-sided surfaces

Model 503/58 a is an rectangular band, the two small sides of which are connected after torsion through an angle of 180°. This so-called **Moebius band** is the most simple example of a one-sided surface.

The **models 504/58 b, 505/58 c, 506/58 d** show **a one-sided ruled surface,** generated by the movement of a straight line. In **model 504/58 b** a circle k and a segment $\overline{AB}$ are given; $\overline{AB}$ is parallel to the axis of the circle and its centre M is situated on k (starting position). The plane through the axis of the circle rotates about this axis, and simultaneously $\overline{AB}$ rotates in this same plane about M such that the angle of the first rotation is twice the angle of the second rotation.

Model 505/58 c represents that one-sided ruled surface, which results from model 504/58 b by substituting the segment $\overline{AB}$ through the straight line AB. In this model those parts of the generating lines are represented as one receives by extending the segments $\overline{AB}$ of 504/58 b to the curve of self-penetration. This one-sided ruled surface has the following parametric equations. If φ denotes the angle of rotation of the segment $\overline{AB}$, r the radius of k, and λ the distance from M of a point P of AB,

$$x = (r - \lambda \sin \varphi) \cos 2\varphi,$$
$$y = (r - \lambda \sin \varphi) \sin 2\varphi,$$
$$z = \lambda \cos \varphi,$$

are the coordinates of P in a certain Cartesian coordinate system, the z-axis of which coincides with the axis of k, its origin is the centre of k.

Model 506/58 d shows the same one-sided ruled surface as model 504/58 b. Here the generating lines are limited by the lateral area of a cylinder, coaxial to the axis of the director circle k, and the planes of its two base circles.