

of points are chosen and corresponding points are united by straight lines, the correspondance being such, that the z-axis belongs to that latter family of lines. This family of lines generates a hyperbolic paraboloid, one ruling plane of which runs parallel to the chosen skew lines. Rotating about the z-axis the latter lines generate planes; the family of the above mentioned uniting lines generates a family of one-sheet hyperboloids of revolution, a so-called osculating pencil.

Model 920/202 a shows the meridian curve of a surface of revolution. Those parts of the meridian that generate elliptic points of the surface are coloured green, those that generate hyperbolic points are marked red. Points in which the tangents of the meridian are parallel to the axis of revolution generate equator circles or gorge circles according as these points are elliptic or hyperbolic ones. The generating points of these circles are marked by green or red balls respectively.

Along the equator and gorge circles cylinders of revolution can be circumscribed the surface of revolution. The transition between the elliptic and the hyperbolic points is generally brought about by parabolic points. In a meridian these are usually points of inflexion or extrema (marked by white balls).

Model 921/202 c. Torus or Ring surface

The surface of revolution of a circle of radius b about a line in the plane of the circle at a distance a from the centre of the circle is called a torus or ring surface; this is an algebraic surface of the degree four with the equation

$$(x^2 + y^2 + z^2 + a^2 - b^2) = 4a^2 (x^2 + y^2)$$

in a suitably chosen Cartesian coordinate system.

We know three types of ring surfaces:

- | | | |
|----|---------|------------------------|
| a) | $a > b$ | an open torus, |
| b) | $a < b$ | an intersecting torus, |
| c) | $a = b$ | a closed torus. |

In the case of the open torus the highest and the lowest parallel circle (parabolic points) separate the regions of the hyperbolic points from the region of the elliptic points.

From the equation of the curve it is to be seen that it has the absolute spherical circle as double curve.

Model 922/202 d. Intersecting torus

The intersecting torus has singular points, they are "conical points" (these points are marked by little blue balls). The transition from the elliptic points to the hyperbolic ones may be brought about also by these conical points.
