Demonstratio. Sit $\sqrt[m.n]{A} = \alpha$, erit $\alpha^{m.n} = A$ (§. 6.), atque hinc $\sqrt[m]{\left(\sqrt[n]{A}\right)} = \sqrt[m]{\left(\sqrt[n]{\alpha^{m.n}}\right)} = \sqrt[m]{\left(\alpha^{m}\right)} = \alpha$ (§.6. et §.22).

Quum vero sit ex hypothesi $\sqrt[m.n]{A} = \alpha$, atque per demonstr. $\sqrt[m]{\left(\sqrt[n]{A}\right)} = \alpha$, est quoque $\sqrt[m]{\left(\sqrt[n]{A}\right)} = \sqrt[m.n]{A}$. Q. E. D.

§. 27. *Coroll.*

Proinde $\sqrt[m]{.}\sqrt[n]{.}\sqrt[p]{A} = \sqrt[m.n.p]{A}$, et vice versa, si sit $q = \alpha . \beta . \gamma$ producto e numeris numerantibus integris, erit $\sqrt[q]{A} = \sqrt[\alpha]{.}\sqrt[\beta]{.}\sqrt[\gamma]{A}$, et s. p.

§. 28. *Theorema.*

$$\left(A^{\frac{\pm n}{m}}\right)^{\pm p} = A^{\frac{\pm n \times \pm p}{m}}$$

Demonstratio. Est enim $A^{\frac{\pm n}{m}} = \sqrt[m]{A^{\pm n}}$ (§.13.) $= \left(\sqrt[m]{A}\right)^{\pm n}$ (§.23), proinde $\left(A^{\frac{\pm n}{m}}\right)^{\pm p} = \left(\left(\sqrt[m]{A}\right)^{\pm n}\right)^{\pm p} = \left(\sqrt[m]{A}\right)^{\pm n \times \pm p}$ (§.21) $= \sqrt[m]{A^{\pm n \times \pm p}}$ (§.23.) $= A^{\frac{\pm n \times \pm p}{m}}$ (§.13). Q. E. D.